# Math708 - Homework 4

1. Determine the quadrature points and weights for the weight function $w(x) = -\ln x$ on the interval (0,1), for $n = 0$ and $n = 1$.

2. The $n$-point Gauss-Lobatto quadrature rule ($n > 1$) is the rule $\int_{-1}^{1} f dx \approx \sum_{i=1}^{n} w_i f(x_i)$ where the $x_1 = -1, x_n = 1$, and the other nodes and weights are chosen so that the degree of precision is as high as possible. Determine the rule for $n = 2, 3$, and 4.

3. Let $f : R \to R$ be a $C^2$ function with a root $x_*$ such that neither $f'$ nor $f''$ has a root. Prove that Newton's method converges to $x_*$ for any initial guess $x_0 \in R$.

4. (Computer Exercise) Apply Adaptive Quadrature with Simpson's rule to solve

$$\int_{-\pi}^{\pi} \cos(x) e^{x^2} dx.$$

   with error tolerance $10^{-5}$.

5. (Computer Exercise) Find the root of the equation

$$2x(1 - x^2 + x) \ln x = x^2 - 1$$

   in the interval $[0, 1]$ by Newton's method. Vary initial guess $x_0$, and make a table that shows the number of correct digits in each step.