Math708 - Homework 1

1. Let $f(x) = e^x, I = [0, 1]$. For $p = 1, 2$ and ∞, find the best L^p approximation to f in $P_0(I)$.

2. a: Construct Lagrange's interpolation polynomial for the data given below.

   b: Construct Newton's interpolation polynomial for the data shown. Without simplifying it, write the polynomial obtained in nested form for easy evaluation.

x	0	2	3	4
y	7	11	28	63

3. Prove that $p_n(x) - p_{n-1}(x) = c(x - x_0)(x - x_1)\cdots(x - x_{n-1})$ for some constant c. We use the notation $f[x_0, x_1, \cdots, x_n]$ to denote this constant and call it the *nth divided difference* of f at the x_i. Use the Lagrange's formula for the inerpolating polynomia to derive an expression for $f[x_0, x_1, \cdots, x_n]$ in terms of x_i and $f(x_i)$.

4. (Computer Exercise) Using n equally spaced nodes (and n Chebyshev nodes) on the interval $[-5, 5]$, find the interpolating polynomial p of degree n for the function $f(x) = (x^2 + 1)^{-1}$. Plot two functions with different values of n ($n = 5, 11, 21, 41$), and observe the discrepancy between $f(x)$ and $p(x)$.