

Optimization

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Overview

The methods learned in Chapter 4 (of Stewart) for finding extreme values have practical applications in many areas of life. In this lab, you will use Maple to help solving several optimization problems.

Maple Essentials

- The new Maple commands introduced in this lab are:

| Command | Description | Example |
|-------------------------------|---|--|
| <code>solve</code> | solves one or more equations for their unknowns | <code>solve(2*x+5*y-7=0, {y});</code> |
| <code>with(RealDomain)</code> | excludes complex number solutions* | <code>solve(x^3=8, {x});</code> <code>with(RealDomain):</code> <code>solve(x^3=8, {x});</code> |
| <code>f:=(a,b)-></code> | multivariate function definition | <code>A:=(r, h)-></code> <code>2*Pi*r^2+2*Pi*r*h;</code> |

***Note:** By default, Maple performs computations under the assumption that the underlying system is the complex field. The `RealDomain` package provides an environment in which computations are performed under the assumption that the basic underlying number system is the field of real numbers.

Related course material/Preparation

§4.7 of the Calculus Text and §5.2, §5.5, and §5.6 of the Maple Text.

General Steps for Solving Optimization Problems

The following strategy for solving optimization problems is outlined on Page 322 of Stewart.

- Understand the problem.
(What is the unknown? What are the given quantities and conditions?)
- Draw a diagram. In most problems it is useful to draw a diagram and identify the given and required quantities on the diagram.
- Introduce notation. Assign a symbol for the quantity, Q , that is to be maximized or minimized. Also select symbols for other unknown quantities. Use suggestive notation whenever possible: A for area, h for height, r for radius, etc.
- Express Q in terms of some of the other unknowns.
- If Q has been expressed as a function of more than one variable, use the given information to find relationships (in the form of equations) among these variables. Then use the equations to eliminate all but one of the variables in the expression for Q .
- Use the methods of Sections 4.1 and 4.3 to find the *absolute* maximum or minimum value.

Activities

Solve each of the optimization problems on this page using Maple when necessary.

Example Problem

We will solve Example 1 on page 322 of Stewart together using Maple:

A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

Steps:

- Optimization problems often use the same variables, so you should start every new problem with `restart`. Also, you will want to load the Maple `RealDomain` package to avoid complex solutions.


```
> restart;
> with(RealDomain):
```
- Assign our given information about the perimeter using `:=`.


```
> perimeter:= 2*x+y=2400;
```
- Use the multivariate function definition to input an equation for area (the quantity to be optimized).


```
> A:= (x,y)-> x*y;
```
- We use the perimeter equation to solve for one of the variables and substitute to get area as a function of only one variable.


```
> solve(perimeter, y);
> y:=x->(Insert expression);
> A(x,y(x));
> A:=x-> (Insert expression);
```
- We find (and assign) the derivative so we can find the critical points.


```
> diff(A(x), x);
> dA:=x->(Insert label);
> solve(dA(x)=0, {x});
```

 Remember you can use `assign` with a label to quickly assign the value(s).
- For this problem, we can check the area at critical point(s) and the endpoints to show that we have the absolute maximum area possible. Enter appropriate values for a , b , and c below.


```
> A(a);
> A(b);
> A(c);
```
- What are the dimensions of the field that has the largest area?

Additional Problems

- A rectangular storage container with an open top is to have a volume of $10m^3$. The length of its base is twice the width. Material for the base costs \$10 per square meter. Material for the sides costs \$6 per square meter. Find the cost of materials for the cheapest such container. (Ex. 14, Stewart page 328)
- Find the point on the parabola $y^2 = 2x$ that is closest to the point $(1, 4)$. (Ex. 3, Stewart page 325)
- A man launches his boat from a point A on the bank of a straight river, 3 km wide, and wants to reach point B , 8 km downstream on the opposite bank, as quickly as possible. He could row his boat directly across the river to point C and then run to B , or he could row directly to B , or he could row to some point D between C and B and then run to B . If he can row 6 km/hr and run 8 km/hr, where should he land to reach B as soon as possible? (Ex. 4, Stewart page 325)

Assignment

Complete lab activities and review Lab 7 to Lab 11 for Lab Quiz 2 next week.