Math 172 Fall 2012 Worksheet 6

1.

$$A \cdot u = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

which is not proportional to u (since $\dfrac{6}{1} \neq \dfrac{3}{2}$), so u is not an eigenvector.

$$A \cdot v = \begin{bmatrix} -12 \\ 3 \end{bmatrix} = -3v$$

so v is an eigenvector with corresponding eigenvalue -3 (since $\dfrac{-12}{4} = \dfrac{3}{-1} = -3$).

2. a. $B_1 = 6 * (1.4)v_1 - 0.8v_2$, $B_2 = 6 * (1.4)^2 v_1 - (0.8)^2 v_2$

b. $B_n = 6 * (1.4)^n v_1 - (0.8)^n v_2$

c. and d. when n is sufficiently large, the general formula from part **b.** can be rounded off to $B_n = 6 * (1.4)^n v_1$. This gives a total population size of $P_n = 6 * (1.4)^n * (12 + 20) = 192 * (1.4)^n$ (thus the total population size has exponential behavior with per capita growth rate $r = 0.4$) and distribution vectors

$$D_n = \begin{bmatrix} 0.375 \\ 0.625 \end{bmatrix}$$

Since these disribution vectors do not depend on n, they give the value of the stable distribution vector.

3.

A frog population has three stages: tadpoles T_n, juveniles J_n and adults A_n.

Each year, 20% of tadpoles become juveniles and 80% of tadpoles die. There are no tadpoles that remain in the same stage at the next step. Also, 70% of juvenile become adults and 30% of juveniles die. There are no juveniles that remain in the same stage. 55% of adults survive, the rest die.

On average each adult produces 40 tadpoles a year. The tadpoles and juveniles don't reproduce.

a. 14%; 0.7

b. 38.5%; 21.2%

c. 30.3%; 16.7%

d. transition matrix:

$$A = \begin{bmatrix} 0 & 0 & 40 \\ 0.2 & 0 &) \\ 0 & 0.7 & 0.55 \end{bmatrix}$$

population vector at $t = 20$:

$$B_{20} = \begin{bmatrix} 222,374,036 \\ 21,509,873 \\ 10,559,716 \end{bmatrix}$$

distribution vector at $t = 20$:

$$D_{20} = \begin{bmatrix} 0.874 \\ 0.085 \\ 0.041 \end{bmatrix}$$

e. The values for the total size of the population at times $t = 20, t = 21, t = 22$ are: $P_{20} = 254,443,625; P_{21} = 487,728,182; P_{22} = 961,675,883$. The ratios are

$$\frac{P_{22}}{P_{21}} = 1.972 \neq \frac{P_{21}}{P_{20}} = 1.917$$

Since the ratios are not the same we conclude that the population does not yet have exponential behavior at $t = 21$ (perhaps one should explore what happens further, but I will accept this kind of answer as correct).