Math 172 Fall 2012 Worksheet 6

1. We are given the following matrix and vectors:

$$A = \begin{bmatrix} -2 & 4 \\ 1 & 1 \end{bmatrix} \quad u = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad v = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

Is u an eigenvector for the matrix A? Justify your answer. If yes, what is the corresponding eigenvalue?

Is v an eigenvector for the matrix A? Justify your answer. If yes, what is the corresponding eigenvalue?

2. A population is divided into two age classes and the transition matrix A has eigenvalues $\lambda_1 = 1.4$ and $\lambda_2 = 0.8$. The corresponding eigenvectors are v_1 and v_2 and the initial population vector is $B(0) = 6v_1 - v_2$.

a. Express $B(1)$ and B_2 in terms of v_1 and v_2.

b. Express B_n in terms of v_1 and v_2.

c. We are given

$$v_1 = \begin{bmatrix} 12 \\ 20 \end{bmatrix} \quad v_2 = \begin{bmatrix} 4 \\ 15 \end{bmatrix}$$

Use this information to find the stable distribution vector that is eventually reached when the population reaches a stable state.

d. Describe the exponential behavior of the total population in the long run.

3. A frog population has three stages: tadpoles T_n, juveniles J_n and adults A_n.

Each year, 20% of tadpoles become juveniles and 80% of tadpoles die. There are no tadpoles that remain in the same stage at the next step. Also, 70% of juvenile become adults and 30% of juveniles die. There are no juveniles that remain in the same stage. 55% of adults survive, the rest die.

On average each adult produces 40 tadpoles a year. The tadpoles and juveniles don't reproduce.

a. What is the probability for a tadpole to survive for two consecutive years (until it becomes adult)? Three consecutive years?

b. What is the probability for a juvenile to survive for two consecutive years? Three consecutive years?

c. What is the probability for an adult to survive for two consecutive years? Three consecutive years?

d. Write the transition matrix and compute the population vector and distribution vector at $t = 20$ if the initial population consists of 100 tadpoles, 20 juveniles, and 20 adults.

e. Does the population have exponential behavior in the long run? Present numerical evidence for your answer and state what is the per capita growth rate r.