Extra Credit Problems

1. a. Give an example of a continuous model affine equation that has an unstable equilibrium value of 100.

b. Describe a real-life process that could be modeled by your equation.

c. Use the method of separation of variables to solve your equation. Assume an initial value of 200. Show work.

d. Use the solution you find in d. to estimate how long it will take for the population to reach 400.

2. A population is divided into four age classes and it has transition matrix

$$A = \begin{bmatrix} 0 & 1 & 6 & 0 \\ 0.3 & 0.2 & 0 & 0 \\ 0 & 0.4 & 0.8 & 0 \\ 0 & 0 & 0.1 & 0.7 \end{bmatrix}$$

The initial population vector is

$$B_0 = \begin{bmatrix} 5 \\ 5 \\ 5 \\ 5 \end{bmatrix}$$

a. Describe the transition from one step to the next in words (that is, describe how many individuals remain in the same age category, how many move from one age class to the next, how many die from each age class; also how many new individuals are born).

b. Find a value of n that is sufficiently large for the population to reach a stable state. What is the stable distribution vector that is reached?

c. Describe the exponential behavior of the population when n is large enough.

3. Two populations N_1, N_2 are in comptetition with each other. We are assuming that N_1 follows a logistic model with Alee effect when N_2 is not present. When N_1 is not present, N_2 follows a standard logistic model. The equations are

$$\frac{dN_1}{dt} = 0.1N_1 \frac{(100 - N_1)(N_1 - 20) - 5N_2}{2000}$$

$$\frac{dN_2}{dt} = 0.1N_2 \frac{160 - N_2 - 2N_1}{160}$$

a. What are the carrying capacitites of the two populations?

b. What are the equations of the isoclines? Draw the isoclines.

c. Find the equilibrium value with both populations present (warning: this requires more advanced algebra than what we did in class)

d. Show arrows to indicate the behavior of the two populations in each one of the four regions of the state space. Explain your reasoning. (warning: this model is different from the one we studied in class; you need to figure out the direction of the arrows based on whether the derivaties are positive or negative in each case).