Answers to WS9

1. a. $\dfrac{du}{dt} = .4u$ – exponential growth

b. $\dfrac{dv}{dt} = -.6v$ – exponential decline

c. u =victims, v = predators

d. $v = 50$

e. $u = 60$.

f. equilibrium: $(0, 0)$, $(60, 50)$. state space: the isoclines are a horizontal line $v = 50$ and a vertical line $u = 60$. the arrows indicating short term behavior point in counterclockwise direction around the equilibrium at $(60, 50)$

(in the region above $v = 50$ and to the right of $u = 60$ the arrow points up and to the left; in the region above $v = 50$ and to the left of $u = 60$ the arrow points down and to the left; in the region below $v = 50$ and to the left of $u = 60$ the arrow points down and to the right; in the region below $v = 50$ and to the right of $u = 60$ the arrow points up and to the right).

2.

the plot for a. should show spiraling outward

the plots for b. and c. should show oscillations above and below the equilibrium values (60 for u, 50 for v).

d. The points $(u(n), v(n))$ spiral outward farther and farther from the equilibrium. Eventually one or both of the species becomes extinct (when the trajectory hits one of the axis of coordinates).

3. a. If $P = 0$, the equation of V is

$$\frac{dv}{dt} = 4V - .06V^2 = 4V\left(1 - \frac{V}{66.7}\right)$$

logistic equation with carrying capacity $K = 66.7$.

If $V = 0$ then the equation of P is

$$\frac{dP}{dt} = -6P$$

exponential decline

b. if $V = 0$ then P will approach zero due to the exponential decline behavior.

If $P = 0$ then V will approach 66.7 due to the logistic behavior. Thus the equiibrium points with at most one of the populations present are $(0, 0)$ and $(66.7, 0)$.

c. $(60, .5)$.

d. the value of V at equilibrium is smaller than the carrying capacity of V. this is due to the fact that the presence of the predators inhibits

the growth of V causing it to not be able to reach its full carrying capacity.

4. there is no equilibrium with both populations present. This is due to the fact that the value of V required to maintain the P population stable is $V = 60$, which is larger that the carrying capacity $K = 40$. Thus the victim population cannot achieve the level needed to sustain the predator population.