1. Without solving the integral, find the appropriate change of variables and simplify the integral.

(a) $\displaystyle\int \left(36 - 9x^2\right)^{5/2}\, dx$

$1 - \sin^2\theta = \cos^2\theta$

(b) $\displaystyle\int \frac{1}{\sqrt{x^2+20}}\, dx$

$\tan^2\theta + 1 = \sec^2\theta$

(c) $\displaystyle\int \sqrt{x^2+2x+10}\, dx$

Complete the square and $\tan^2\theta + 1 = \sec^2\theta$

(d) $\displaystyle\int \left(9 - 4t^2\right)^{3/2}\, dt$

$1 - \sin^2\theta = \cos^2\theta$

(e) $\displaystyle\int \frac{x^2+4}{\sqrt{x^2-4}}\, dx$

$\sec^2\theta - 1 = \tan^2\theta$

(f) $\displaystyle\int e^{4x}\sqrt{e^{8x}-9}\, dx$

Do u-substitution and $\sec^2\theta - 1 = \tan^2\theta$

(g) $\displaystyle\int \frac{z}{\left(1-4z-2z^2\right)^{3/2}}\, dz$

Complete the square and $1 - \sin^2\theta = \cos^2\theta$

(h) $\displaystyle\int \frac{1}{\sqrt{9x^2-36x+37}}\, dx$

Complete the square and $\tan^2\theta + 1 = \sec^2\theta$

(i) $\displaystyle\int \sqrt{7w^2-1}\, dw$

$\sec^2\theta - 1 = \tan^2\theta$

(j) $\int \cos(x)\sqrt{9+25\sin^2(x)}\,dx$

Do u-substitution and $\tan^2\theta + 1 = \sec^2\theta$

(k) $\int \frac{1}{\sqrt{10x - x^2}}\,dx$

Complete the square and $1 - \sin^2\theta = \cos^2\theta$

2. Evaluate the following integrals

(a) $\int \frac{4}{\sqrt{1-x^2}}\,dx$

Use $x = \sin\theta,\quad dx = \cos\theta\,d\theta,\quad 1 - \sin^2\theta = \cos^2\theta$

(b) $\int \frac{1}{1+x^2}\,dx$

Use $x = \tan\theta,\quad dx = \sec^2\theta\,d\theta,\quad \tan^2\theta + 1 = \sec^2\theta$

(c) $\int \frac{1}{\sqrt{9-x^2}}\,dx$

Use $x = 3\sin\theta,\quad dx = 3\cos\theta\,d\theta,\quad 1 - \sin^2\theta = \cos^2\theta$

(d) $\int \frac{x}{\sqrt{x^2+1}}\,dx$

Use $x = \tan\theta,\quad dx = \sec^2\theta\,d\theta,\quad \tan^2\theta + 1 = \sec^2\theta$

(e) $\int \frac{x^3}{\sqrt{16-x^2}}\,dx$

Use $x = 4\sin\theta,\quad dx = 4\cos\theta\,d\theta,\quad 1 - \sin^2\theta = \cos^2\theta$

(f) $\int \frac{1}{x^2\sqrt{25-x^2}}\,dx$

Use $x = 5\sin\theta,\quad dx = 5\cos\theta\,d\theta,\quad 1 - \sin^2\theta = \cos^2\theta$

(g) $\int \frac{x^3}{\sqrt{64+x^2}}\,dx$

Use $x = 8\tan\theta,\quad dx = 8\sec^2\theta\,d\theta,\quad 1 + \tan^2\theta = \sec^2\theta$

(h) $\displaystyle\int \sqrt{x^2+1}\,dx$

Use $x = \tan\theta,\quad dx = \sec^2\theta\,d\theta,\quad \tan^2\theta + 1 = \sec^2\theta$

(i) $\displaystyle\int x^2\sqrt{4-x^2}\,dx$

Use $x = 2\sin\theta,\quad dx = 2\cos\theta\,d\theta,\quad 1-\sin^2\theta = \cos^2\theta$

Use $\sin^2\theta = \dfrac{1-\cos(2\theta)}{2}$ and $\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$

(j) $\displaystyle\int \frac{x^2}{\sqrt{1-x^2}}\,dx$

Use $x = \sin\theta,\quad dx = \cos\theta\,d\theta,\quad 1-\sin^2\theta = \cos^2\theta$

(k) $\displaystyle\int \frac{x^2}{\sqrt{4+x^2}}\,dx$

Use $x = 2\tan\theta,\quad dx = 2\sec^2\theta\,d\theta,\quad 1+\tan^2\theta = \sec^2\theta$

(l) $\displaystyle\int \frac{3x-1}{x^2+1}\,dx$

Use $x = \tan\theta,\quad dx = \sec^2\theta\,d\theta,\quad 1+\tan^2\theta = \sec^2\theta$

(m) $\displaystyle\int \frac{(1-x^2)^{3/2}}{x^2}\,dx$

Use $x = \sin\theta,\quad dx = \cos\theta\,d\theta,\quad 1-\sin^2\theta = \cos^2\theta$

(n) $\displaystyle\int \frac{\sqrt{1-x^2}}{x}\,dx$

Use $x = \sin\theta,\quad dx = \cos\theta\,d\theta,\quad 1-\sin^2\theta = \cos^2\theta$

(o) $\displaystyle\int \frac{1}{\sqrt{x^2-81}}\,dx$

Use $x = 9\sec\theta,\quad dx = 9\sec\theta\tan\theta\,d\theta,\quad \sec^2\theta - 1 = \tan^2\theta$

(p) $\displaystyle\int_0^1 \frac{1}{\sqrt{x^2+1}}\,dx$

Use $x = 4\tan\theta, \quad dx = 4\sec^2\theta\,d\theta, \quad \tan^2\theta + 1 = \sec^2\theta$

(q) $\displaystyle\int_0^5 \sqrt{10x - x^2}\,dx$

Complete the square and use $x = 5\,(\sin\theta + 1), \quad dx = 5\cos\theta\,d\theta, \quad 1 - \sin^2\theta = \cos^2\theta$