Worksheet - Trigonometric integrals

1. What are the three Pythagorean trigonometric identities?

The **only one** you need to remember is $\sin^2 x + \cos^2 x = 1.$

2. What are the power reduction (half angle) formulas for $\sin^2 x$ and $\cos^2 x$?

For the half-angle and double angle formulas we just need to remember the following two identities

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

3. Evaluate the following integrals

(a). $\displaystyle\int \sin^4 x\, dx$

Use $\sin^2 x = \dfrac{1-\cos(2x)}{2}$ and $\cos^2(2x) = \dfrac{1+\cos(4x)}{2}$

(b). $\displaystyle\int \frac{\sin x}{\cos^3 x}\, dx$

Use $u = \cos x \;\Rightarrow\; du = -\sin x\, dx$

(c). $\displaystyle\int \cos^2 x\, dx$

Use $\cos^2 x = \dfrac{1+\cos(2x)}{2}$

(d). $\displaystyle\int \sin^3 x \cos^2 x\, dx$

Use $u = \cos x \;\Rightarrow\; du = -\sin x\, dx$

(e). $\displaystyle\int_0^{\pi/2} \sin^2 x \cos^2 x\, dx$

Use $\cos^2 x = \dfrac{1+\cos(2x)}{2}$ and $\sin^2 x = \dfrac{1-\cos(2x)}{2}$

(f). $\displaystyle\int \sin^3 x\, dx$

Use $u = \cos x \;\Rightarrow\; du = -\sin x\, dx$

(g). $\displaystyle\int x \sec^2(x^2)\tan^4(x^2)\, dx$

Use $u = x^2 \;\Rightarrow\; du = 2\,x\,dx$ and $v = \tan u \;\Rightarrow\; dv = \sec^2 u\, du$

(h). $\int \tan^2 x \, dx$

Use $\tan^2 x = \sec^2 x - 1$

(i). $\int \tan^5 x \sec^3 x \, dx$

Use $\tan^2 x = \sec^2 x - 1$ and $\frac{d}{dx}(\sec(x)) = \sec x \tan x$

(j). $\int \sin(8x) \cos(5x) \, dx$

With this type of integrals we integrate by parts *twice* and rearrange terms.

(k). $\int x^2 \sin x \, dx$

Use integration by parts and tabulate

(l). $\int \frac{1 - \tan^2 x}{\sec^2 x} \, dx$

Use $\tan^2 x = \sec^2 x - 1$ and $\cos^2 x = \frac{1 + \cos(2x)}{2}$

(m). $\int_{\pi/4}^{\pi/2} \cot^3 x \, dx$

Use $\cot^2 x = \csc^2 x - 1$ and separate into two integrals.

(n). $\int \sec^2 x \, dx$

Use $\frac{d}{dx} \tan x = \sec^2 x$

(o). $\int \csc x \cot x \, dx$

Use $\frac{d}{dx} \csc x = -\csc x \cot x$