

1. Find the Maclaurin series for $f(x)$ and the associated radius of convergence

(a) $f(x) = (1 - x)^{-2}$

(b) $f(x) = \cos(3x)$

(c) $f(x) = x e^x$

(d) $f(x) = e^{5x}$

2. Find the Taylor series for $f(x)$ centered at the given value of a .

(a) $f(x) = x^4 - 3x^2 + 1, \quad a = 1$

(b) $f(x) = x^{-2}, \quad a = 1$

(c) $f(x) = \cos x, \quad a = \pi$

(d) $f(x) = \frac{1}{x}, \quad a = -3$

3. Use series to approximate the definite integral.

(a) $\int_0^1 x \cos(x^3) dx$

(b) $\int_0^1 x^2 e^{-x^2} dx$

4. Use series to evaluate the limit

(a) $\lim_{x \rightarrow 0} \frac{x - \tan^{-1} x}{x^3}$

(b) $\lim_{x \rightarrow 0} \frac{\sin x - x + \frac{1}{6}x^3}{x^5}$

5. Use Taylor's Inequality to determine the number of terms of the Maclaurin series for $f(x) = e^x$ that should be used to estimate $e^{0.1}$ to within 0.00001 error.

6. A car is moving with speed 20 m/s and acceleration 2 m/s² at a given instant. Using a second-degree Taylor polynomial, estimate how far the car moves in the next second. Would it be reasonable to use this polynomial to estimate the distance traveled during the next minute?

7. An electric dipole consists of two electric charges of equal magnitude and opposite sign. If the charges are q and $-q$ and are located at a distance d from each other, then the electric field E at the point P in the figure is

$$E = \frac{q}{D^2} - \frac{q}{(D + d)^2}.$$

By expanding this expression for E as a series in powers of d/D , show that E is approximately proportional to $1/D^3$ when P is far away from the dipole.



8. If a surveyor measures differences in elevation when making plans for a highway across a desert, corrections must be made for the curvature of the earth. If the radius of the earth is R and L is the length of the highway, it can be shown that the correction is given by

$$C = R \sec\left(\frac{L}{R}\right) - R$$

Use a Taylor polynomial to show that

$$C \approx \frac{L^2}{2R} + \frac{5L^4}{24R^3}.$$