Worksheet - Power Series

1. Find the radius of convergence and interval of convergence of the series

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n+1}$

   (c) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$

   (d) $\displaystyle\sum_{n=1}^{\infty} n^n\, x^n$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{5^n n^5}$

   (f) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{4^n}(x+1)^n$

   (g) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^n}$

   (h) $\displaystyle\sum_{n=1}^{\infty} \frac{n\,(x-4)^n}{n^3+1}$

2. Suppose that $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = -4$ and diverges when $x = 6$. What can be said about the convergence or divergence of the following series.

   (a) $\displaystyle\sum_{n=0}^{\infty} c_n\, 8^n$

   (b) $\displaystyle\sum_{k=0}^{\infty} (-1)^n c_n\, 9^n$

3. Suppose that $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = 2$ and diverges when $x = -3$. What can be said about the convergence or divergence of the following series.

   (a) $\displaystyle\sum_{k=0}^{\infty} \frac{4^n}{5^{n+1}} c_n$

   (b) $\displaystyle\sum_{n=0}^{\infty} c_n\,(-3)^n$

4. Find a power series representation for the function and determine the interval of convergence

   (a) $f(x) = \dfrac{1}{1+x}$

   (b) $f(x) = \dfrac{3}{1-x^4}$

   (c) $f(x) = \dfrac{x^2}{a^3-x^3}$

   (d) $f(x) = \ln(5-x)$

   (e) $f(x) = \ln(x^2+4)$

   (f) $f(x) = \dfrac{2x}{(1-x^2)^2}$

   (g) $f(x) = \tan^{-1}(x^2)$

5. Evaluate the integral as a power series. What is the radius of convergence?

   (a) $\displaystyle\int \frac{t}{1-t^8}\, dt$

   (b) $\displaystyle\int \frac{\ln(1-t)}{t}\, dt$

   (c) $\displaystyle\int \frac{x^2}{1+x^4}\, dx$

6. Use partial fractions to find the power series of each of the following functions.

   (a) $\dfrac{4}{(x-3)(x+1)}$

   (b) $\dfrac{5}{(x^2+4)(x^2-1)}$

   (c) $\dfrac{3}{(x+2)(x-1)}$