

Integration by parts formula

$$\int u dv = uv - \int v du$$

Find the following integrals

1. $\int x e^x dx$ Use $u = x \Rightarrow du = dx$ $dv = e^x dx \Rightarrow v = e^x$

2. $\int \sqrt{x} \ln x dx$ Use $u = \ln x \Rightarrow du = \frac{1}{x} dx$ $dv = \sqrt{x} dx \Rightarrow v = \frac{2}{3} x^{3/2}$

3. $\int t \cdot 2^t dt$ Use $u = t \Rightarrow du = dt$ $dv = e^{t \ln 2} dt \Rightarrow v = \frac{1}{\ln 2} e^{t \ln 2}$

4. $\int x \cos x dx$ Use $u = x \Rightarrow du = dx$ $dv = \cos x dx \Rightarrow v = \sin x$

5. $\int t^2 \cos(3t) dt$ Use $u = t^2 \Rightarrow du = 2t dt$ $dv = \cos(3t) dt \Rightarrow v = \frac{1}{3} \sin(3t)$

6. $\int x e^{-x} dx$ Use $u = x \Rightarrow du = dx$ $dv = e^{-x} dx \Rightarrow v = -e^{-x}$

7. $\int x^3 \ln x dx$ Use $u = \ln x \Rightarrow du = \frac{1}{x} dx$ $dv = x^3 dx \Rightarrow v = \frac{1}{4} x^4$

8. $\int x^2 e^{6x} dx$ Use $u = x^2$ $dv = e^{6x} dx$ and tabular scheme

9. $\int \frac{x^2}{e^{3x}} dx$ Use $u = x^2$ $dv = e^{-3x} dx$ and tabular scheme

10. $\int_0^{\pi/4} \sin x \ln |\cos x| dx$ Use $u = \ln |\cos x| \Rightarrow du = -\frac{1}{\cos x} \sin x dx$ $dv = \sin x dx \Rightarrow v = -\cos x$

11. $\int x \ln x dx$ Use $u = \ln x \Rightarrow du = \frac{1}{x} dx$ $dv = x dx \Rightarrow v = \frac{1}{2} x^2$

12. $\int x^3 \sqrt{4-x^2} dx$ Use $u = x^2 \Rightarrow du = 2x dx$ $dv = x \sqrt{4-x^2} dx \Rightarrow$
 $v = \int x \sqrt{4-x^2} dx = -\frac{1}{2} \int (-2x)(4-x^2)^{1/2} dx = \left(\frac{1}{2}\right) \left(\frac{2}{3}\right) (4-x^2)^{3/2} = -\frac{1}{3} (4-x^2)^{3/2}$

13. $\int x \cos^2 x \, dx$

 Use the trigonometric identity $\cos^2 x = \frac{1 + \cos(2x)}{2}$ and

$$u = x \Rightarrow du = dx \qquad dv = \cos(2x) \, dx \Rightarrow v = \frac{1}{2} \sin(2x)$$

14. $\int e^x \sin x \, dx$

Use

$$u = \sin x \Rightarrow du = \cos x \, dx \qquad dv = e^x \, dx \Rightarrow v = e^x$$

15. $\int 2x \tan^{-1} x \, dx$

Use

$$u = \tan^{-1} x \Rightarrow du = \frac{1}{1+x^2} \, dx \qquad dv = 2x \, dx \Rightarrow v = x^2$$

16. $\int \ln x \, dx$

Use

$$u = \ln x \Rightarrow du = \frac{1}{x} \, dx \qquad dv = dx \Rightarrow v = x$$

17. $\int (2x+1)e^x \, dx$

Use

$$u = 2x+1 \Rightarrow du = 2 \, dx \qquad dv = e^x \, dx \Rightarrow v = e^x$$

18. $\int x^2 \sqrt{x-1} \, dx$

Use

$$u = x^2 \Rightarrow du = 2x \, dx \qquad dv = \sqrt{x-1} \, dx \Rightarrow v = \frac{2}{3}(x-1)^{3/2}$$

19. $\int \sin^{-1} x \, dx$

Use

$$u = \sin^{-1} x \Rightarrow du = \frac{1}{\sqrt{1-x^2}} \, dx \qquad dv = dx \Rightarrow v = x$$

20. $\int (\ln x)^2 \, dx$

Use

$$u = (\ln x)^2 \Rightarrow du = \frac{2 \ln x}{x} \, dx \qquad dv = dx \Rightarrow v = x$$

21. $\int \tan^{-1} x \, dx$

Use

$$u = \tan^{-1} x \Rightarrow du = \frac{1}{1+x^2} \, dx \qquad dv = dx \Rightarrow v = x$$

22. $\int x^3 \sqrt{9-x^2} \, dx$

Use

$$u = \sqrt{9-x^2} \Rightarrow du = -\frac{x}{\sqrt{9-x^2}} \, dx \qquad dv = x^3 \, dx \Rightarrow v = \frac{1}{4}x^4$$

23. $\int e^{2x} \sin x \, dx$

Use

$$u = \sin x \Rightarrow du = \cos x \, dx \qquad dv = e^{2x} \, dx \Rightarrow v = \frac{1}{2} e^{2x}$$

24. $\int \cos^{-1} x \, dx$

Use

$$u = \cos^{-1} x \Rightarrow du = -\frac{1}{\sqrt{1-x^2}} \, dx \qquad dv = dx \Rightarrow v = x$$

25. $\int \frac{\ln x}{x^2} \, dx$

Use

$$u = \ln x \Rightarrow du = \frac{1}{x} \, dx \qquad dv = x^{-2} \, dx \Rightarrow v = -x^{-1} = -\frac{1}{x}$$

26. $\int \sin^3 x \, dx$

Use

$$u = \sin^2 x \Rightarrow du = 2 \sin x \cos x \, dx \qquad dv = \sin x \, dx \Rightarrow v = -\cos x$$

27. $\int x^2 \ln x \, dx$

Use

$$u = \ln x \Rightarrow du = \frac{1}{x} dx$$

$$dv = x^2 dx \Rightarrow v = \frac{1}{3} x^3$$

28. $\int x \sin\left(\frac{x}{2}\right) dx$

Use

$$u = x \Rightarrow du = dx$$

$$dv = \sin\left(\frac{x}{2}\right) dx \Rightarrow v = -2 \cos\left(\frac{x}{2}\right)$$