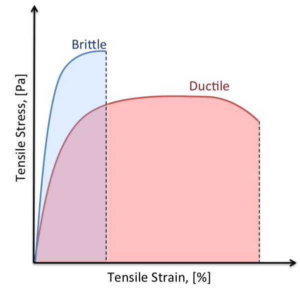
The **toughness** of a material is its ability to absorb energy up to fracture (energy per unit volume of material). The total amount of energy absorption can be calculated by the total area under the curve of a stress-strain tensile test.

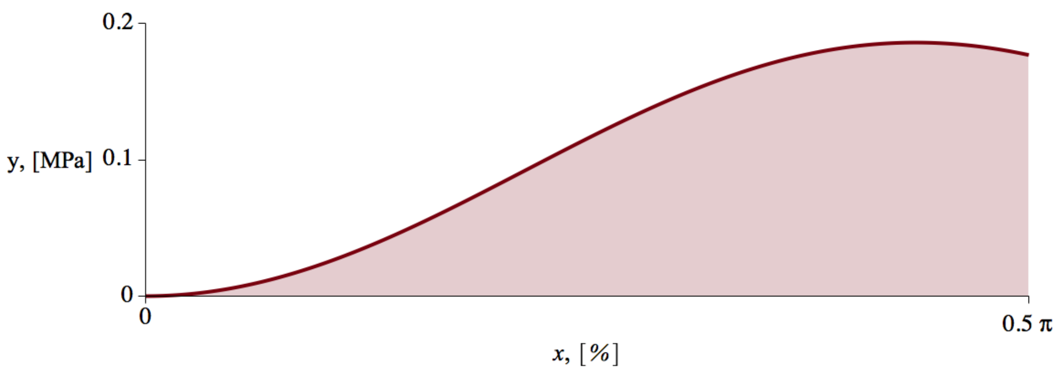


Example: Assume that the stress-strain test curve for a certain material can be approximated as

$$y = \cos^3\left(\frac{\theta}{2}\right)\sin^2\left(\frac{\theta}{2}\right).$$

Find the total energy absorption assuming that breaking occurs for a strain of $\frac{\pi}{2}$%.

The stress-strain curve is shown in the figure below,



to calculate the total energy we need to evaluate the integral

$$\int_0^{0.5\pi} \cos^3\left(\frac{\theta}{2}\right)\sin^2\left(\frac{\theta}{2}\right)d\theta.$$

For this we use the identity $\cos^2\theta = 1 - \sin^2\theta$, which gives

$$\begin{aligned}\int \cos^3\left(\frac{\theta}{2}\right)\sin^2\left(\frac{\theta}{2}\right)d\theta &= \int \cos^2\left(\frac{\theta}{2}\right)\sin^2\theta\cos\left(\frac{\theta}{2}\right)d\theta \\ &= \int\left(1-\sin^2\left(\frac{\theta}{2}\right)\right)\sin^2\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)d\theta,\end{aligned}$$

next we substitute $u = \sin\left(\frac{\theta}{2}\right)$, and $du = \frac{1}{2}\cos\left(\frac{\theta}{2}\right)d\theta$ to obtain

$$\begin{aligned}\int_0^{0.5\pi} \cos^3\left(\frac{\theta}{2}\right)\sin^2\left(\frac{\theta}{2}\right)d\theta &= \int_0^{0.5\pi}\left(1-\sin^2\left(\frac{\theta}{2}\right)\right)\sin^2\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)d\theta \\ &= \int_0^{\sqrt{2}/2}\left(1-u^2\right)u^2\left(2du\right) = 2\int_0^{\sqrt{2}/2}\left(1-u^2\right)u^2du \\ &= 2\left(\frac{u^3}{3}-\frac{u^5}{5}\right|_0^{\sqrt{2}/2} = \frac{70}{60}\sqrt{2}\frac{\text{MJ}}{\text{m}^3}.\end{aligned}$$