$\sum_{k=1}^{\infty} a_k$, here k is the index of summation and the series starts at $k=1$.

This notation is a convenient way of writing $\lim_{n\to\infty} \sum_{k=1}^{n} a_k$.

- **Writing down individual terms:**
  - $$\sum_{i=0}^{\infty} \frac{i-1}{2^{i+2}} = \frac{0-1}{2^{0+2}} + \frac{1-1}{2^{1+2}} + \frac{2-1}{2^{2+2}} + \frac{3-1}{2^{3+2}} + \cdots = -\frac{1}{2^2} + 0 + \frac{1}{2^4} + \frac{2}{2^5} + \cdots$$
  - $$\sum_{n=2}^{\infty} \frac{n-3}{2^{n}} = \frac{2-3}{2^{2}} + \frac{3-3}{2^{3}} + \frac{4-3}{2^{4}} + \frac{5-3}{2^{5}} + \cdots = -\frac{1}{2^2} + 0 + \frac{1}{2^4} + \frac{2}{2^5} + \cdots$$

- **Basic Properties:**
  - If $\sum a_n$ converges, then $\sum c\, a_n$, for any constant c.
  - if $\sum_{n=k}^{\infty} a_n$ and $\sum_{n=k}^{\infty} b_n$ are convergent, then $\sum_{n=k}^{\infty} a_n \pm \sum_{n=k}^{\infty} b_n = \sum_{n=k}^{\infty} (a_n \pm b_n)$ also converges.

  If **one** of them diverges, the sum (or subtraction) also diverges. Note that both series start at the same point.

- **Products of series:** We cannot say anything about convergence from a product of two series since

$$\left(\sum_{n=0}^{\infty} a_n\right)\left(\sum_{n=0}^{\infty} b_n\right) \neq \sum_{n=0}^{\infty} (a_n \cdot b_n)$$

- **Shifting:** To shift the series we use a dummy variable. For example the series $\sum_{n=2}^{\infty} \frac{n-3}{2^n}$, can be shifted to start at zero if we use $i = n-2$, which implies $n = i+2$:

$$\sum_{i=0}^{\infty} \frac{(i+2)-3}{2^{i+2}} = \sum_{i=0}^{\infty} \frac{i-1}{2^{i+2}} = \sum_{n=0}^{\infty} \frac{n-1}{2^{n+2}}$$

Examples

- Write the series $\sum_{n=1}^{\infty} \frac{n^2}{1-3^{n+1}}$ as a series starting at $n=0$

$$n = i+1: \qquad \sum_{n=1}^{\infty} \frac{n^2}{1-3^{n+1}} = \sum_{i=0}^{\infty} \frac{(i+1)^2}{1-3^{(i+1)+1}} = \sum_{n=0}^{\infty} \frac{(n+1)^2}{1-3^{n+2}}$$

- Write the series $\sum_{n=1}^{\infty} \frac{n^2}{1-3^{n+1}}$ as a series starting at $n=3$

$$n = i-2: \qquad \sum_{n=1}^{\infty} \frac{n^2}{1-3^{n+1}} = \sum_{i=3}^{\infty} \frac{(i-2)^2}{1-3^{(i-2)+1}} = \sum_{n=3}^{\infty} \frac{(n-2)^2}{1-3^{n-1}}$$