Definition of a derivative

We are looking for the rate of change of a function at a point,

- Approximate to an interval: dx
- Find the *raw* change: $f(x+dx)-f(x)$
- Find the rate of change on the interval: $\dfrac{f(x+dx)-f(x)}{dx}$
- Approximate to the point: $\dfrac{df(x)}{dx} = f'(x) = \lim_{dx\to 0} \dfrac{f(x+dx)-f(x)}{dx}$

Derivate rules

- Product rule: $(f\cdot g)' = f'\cdot g + f\cdot g'$
- Quotient rule: $\left(\dfrac{f}{g}\right)' = \dfrac{f'\cdot g - f\cdot g'}{g^2}$
- Power rule: $(x^n)' = n\,x^{n-1}$
- Addition/subtraction: $(f\pm g)' = f' \pm g'$
- Multiplication by a constant: $(cf(x))' = c\,(f(x))'$
- Chain rule: $(f\,(g(x)))' = g'(x)\cdot f'\,(g(x))$
- Derivatives of inverse functions: If $f\,(g(x)) = x$, then by the chain rule $g'(x) = \dfrac{1}{f'\,(g(x))}$

Logarithms and exponentials

- $\log(a\cdot b) = \log a + \log b$
- $\log\left(\dfrac{a}{b}\right) = \log a - \log b$ $\quad \log x^a = a\log x \quad a^x = e^{x\log a}$

Integrals

- Indefinite integrals: $\displaystyle\int f(x)dx = F(x) + C,$ where $F'(x) = f(x)$
- Definitive integrals: $\displaystyle\int_a^b f(x)dx = F(b) - F(a)$
- Addition/subtraction: $\displaystyle\int (f\pm g)\,dx = \int f dx \pm \int g dx$
- Multiplication by a constant: $\displaystyle\int cf(x)dx = c\int f(x)dx$