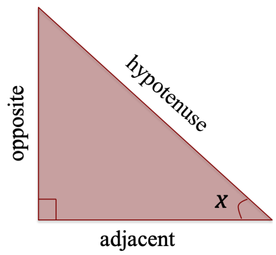
Basics

1. **Basic Definitions**
For the angle x, fill the blanks with *adj* (for adjacent side), *opp* (for opposite side), and *hyp* (for hypothenuse).



$$\sin x = \frac{\square}{\square} \qquad \csc x = \frac{\square}{\square}$$

$$\cos x = \frac{\text{adj}}{\text{hyp}} \qquad \sec x = \frac{\square}{\square}$$

$$\tan x = \frac{\square}{\square} \qquad \cot x = \frac{\square}{\square}$$

2. **Basic Identities**
For the angle x in the figure above, fill the blanks with $\sin x$, $\cos x$, and/or $\tan x$.

$$\tan x = \frac{\sin x}{\square} \qquad \csc x = \frac{\square}{\square}$$

$$\cot x = \frac{1}{\square} = \frac{\square}{\sin x} \qquad \sec x = \frac{\square}{\square}$$

Pythagorean trigonometric identities

The **only one** you need to remember is $\sin^2 x + \cos^2 x = 1.$

The other two can be found by dividing by $\sin^2 x$ *or* $\cos^2 x$.

Find the other two pythagorean identities by following the procedures below.

Dividing by $\sin^2 x$:

$$\sin^2 x + \cos^2 x = 1$$

$$\frac{\sin^2 x}{\square} + \frac{\cos^2 x}{\square} = \frac{1}{\square}$$

$$1 + \left(\frac{\square}{\square}\right)^2 = \left(\frac{\square}{\square}\right)^2$$

$$1 + \square = \square$$

Dividing by $\cos^2 x$:

$$\sin^2 x + \cos^2 x = 1$$

$$\frac{\sin^2 x}{\square} + \frac{\cos^2 x}{\square} = \frac{1}{\square}$$

$$\left(\frac{\square}{\square}\right)^2 + 1 = \left(\frac{\square}{\square}\right)^2$$

$$\square + 1 = \square$$

Power reduction (half angle) formulas for $\sin^2 x$ and $\cos^2 x$

For the half-angle and double angle formulas we just need to remember the following two identities

$$\begin{aligned}\sin(\alpha+\beta) &= \sin\alpha\cos\beta+\cos\alpha\sin\beta \\ \cos(\alpha+\beta) &= \cos\alpha\cos\beta-\sin\alpha\sin\beta\end{aligned}$$

To find the double angle formulas use $\alpha=\beta$ (Simplify as much as possible)

$$\sin(\alpha+\alpha)=\sin(2\alpha) = \sin\alpha\cos\alpha+\cos\alpha\sin\alpha = \boxed{\qquad\qquad\qquad}$$

$$\cos(\alpha+\alpha)=\cos(2\alpha) = \boxed{\qquad\qquad\qquad\qquad\qquad} = \boxed{\qquad\qquad\qquad}$$

To solve for the quadratic form in terms of the double angle, we need to substitute the Pythagorean identity in the expression for $\cos(2\alpha)$.

- To find the expression for $\cos^2(\alpha)$, substitute $\sin^2\alpha = 1-\cos^2\alpha$ in the expression for $\cos(2\alpha)$ and solve for $\cos^2\alpha$

$$\cos^2\alpha = \frac{\boxed{\qquad\qquad\qquad}}{2}$$

- To find the expression for $\sin^2(\alpha)$, substitute $\cos^2\alpha = \boxed{\qquad\qquad}$ in the expression for $\cos(2\alpha)$ and solve for $\sin^2\alpha$

$$\sin^2\alpha = \frac{\boxed{\qquad\qquad\qquad}}{2}$$

Integrals of the form $\int \sin^m x \cos^n x \, dx$

For this kind of integrals the process is simple:

Separate in powers of 2 and look for the odd man out

There are three main cases we consider for this type of integrals:

1. m even and n odd
2. m odd and n even
3. both m and n odd

1. Find the integral $\int \sin^4(x)\cos^3(x)\,dx$

   (a) *Separate in powers of 2 as much as possible*

   $$\int \sin^4(x)\cos^3(x)\,dx = \int \boxed{}$$

   (b) *Determine which function is the odd man out*

   (c) *Once the odd one is identified, determine which substitution is necessary by taking the antiderivative for the function.*

   (d) *Express the integral in terms of the variable used in the substitution*

   (e) *Solve the integral*

   $$\int \left(u^4 - u^6\right)\,du = \boxed{}$$

   (f) *Bring the answer back to the original variable.*

   $$\int \sin^4(x)\cos^3(x)\,dx = \frac{\sin^5 x}{5} - \frac{\sin^7 x}{7} + C.$$

Integrals of the form $\int \sin^m x \cos^n x \, dx$

Separate in powers of 2 and look for the odd man out

2. Find the integral $\displaystyle\int \sin^3(x)\cos^2(x)\,dx$

Integrals of the form $\int \sin^m x \cos^n x \, dx$

Separate in powers of 2 and look for the odd man out

3. Find the integral $\displaystyle\int \sin^5(x)\cos^3(x)\,dx$

Note that in this case, you can choose either function for the substitution.

Integrals of the form $\int \sin^m x \cos^n x \, dx$

The last case for this type of integrals is when both powers are even numbers. In this case the previous approach is of no use.

We separate the integral into powers of 2, but instead of looking for a substitution we use half-angle formulas.

4. Find the integral $\displaystyle\int \sin^2(x)\cos^2(x)\,dx$

Substituting the half-angle formulas gives,

$$\int \left(\sin^2(x)\right)\left(\cos^2(x)\right)\,dx = \int \left(\frac{\boxed{}}{\boxed{}}\right)\left(\frac{\boxed{}}{\boxed{}}\right)dx$$

Simplifying we obtain,

We can apply the half-angle formula one more time to obtain,

$$\frac{\boxed{}}{\boxed{}}\int \left(\frac{1-\cos\left(\boxed{}\right)}{2}\right)dx$$

Finally, we can separate this integral into two integrals,

$$\frac{1}{8}\left[\int \boxed{}\,dx - \int \boxed{}\,dx\right].$$

And solve as,

$$\int \sin^2(x)\cos^2(x)\,dx = \frac{1}{8}\left[x - \frac{1}{4}\sin(4x) + C\right].$$

Integrals of the form $\int \sec^m x \tan^n x \, dx$

Since $\sec x$ and $\tan x$ are somehow related by their derivatives and antiderivatives, we can apply the same process to these integrals. We just need to be a little more careful with the substitutions.

- In general we can always use the identity
$$1 + \tan^2 x = \sec^2 x$$
- To identify the substitution, we need to check whether we have an even or odd power of $\sec x$ left out after we apply the identity.
  - If the power is even, we use the substitution $u = \tan x$, which follows from
  $$\int \sec^2 x \, dx = \tan x + C$$
  - If the power is odd, we use the substitution $u = \sec x \, \tan x$, which follows from
  $$\int \sec x \tan x \, dx = \sec x + C$$

Find the integral $\int \sec^4 x \, \tan^2 \, dx$

- Separate into powers of 2
$$\int \sec^4 x \, \tan^2 \, dx = \int \boxed{}$$
- Since we have only even powers for $\sec x$ we look for $\left(\sec^2 x \, dx\right)$ and since its antiderivative is $\tan x$, we use the substitution $u = \tan x$. Note that in the first parenthesis we used the Pythagorean identity.

- We can solve the integral as
$$\int (u^2 + u^4) \, du = \boxed{}$$
- Finally we need to bring it back to the original variable x to obtain the solution
$$\int \sec^4 x \, \tan^2 \, dx = \frac{1}{3} \tan^3 x + \frac{1}{5} \tan^5 x + C.$$

Integrals of the form $\int \sec^m x \tan^n x \, dx$

Find the integral $\int \sec^3 x \tan x \, dx$.