We have two options:

1. Calculate the series using the formula for Taylor expansion

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = f(z) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \cdots$$

2. Use the series representation of a known function

Function	Maclaurin Series	Interval of Convergence
$f(x) = \frac{1}{1-x}$	$\sum_{n=0}^{\infty} x^n$	$-1 < x < 1$
$f(x) = e^x$	$\sum_{n=0}^{\infty} \frac{x^n}{n!}$	$-\infty < x < \infty$
$f(x) = \sin x$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$	$-\infty < x < \infty$
$f(x) = \cos x$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$	$-\infty < x < \infty$
$f(x) = \ln(1+x)$	$\sum_{n=0}^{\infty} (1-)^{n+1} \frac{x^n}{n}$	$-1 < x \leq 1$
$f(x) = \tan^{-1} x$	$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1}$	$-1 \leq x \leq 1$
$f(x) = (1+x)^r$	$\sum_{n=0}^{\infty} \binom{r}{n} x^n$ $\binom{r}{n} = \frac{r(r-1)(r-2)\cdots(r-n+1)}{n!}$	$-1 < x < 1$

Examples: To find the Taylor expansion of the given function at the given center, decide whether it is better to use option 1 or option 2 above

- $f(x) = \frac{1}{x}$ at $a = 1$

- $f(x) = \cos(2x)$ at $a = \pi$

- $f(x) = \frac{1}{(x-1)^2}$ at $a = 0$

- $f(x) = e^{x^2}$ at $a = 0$

- $f(x) = \frac{x}{4x - 2x^2 - 1}$ at $a = 2$

- $f(x) = x\sin(3x)$ at $a = 0$

Find the 8^{th} Taylor polynomial of $f(x) = \cos(2x)$ at $a = \pi$

n	$f^{(n)}(x)$	$f^{(n)}(\pi)$
0		
1		
2		
3		
4		
5		