A **rational function** is a fraction in which the numerator and denominator are both polynomials. The **degree** of a polynomial is the power of the highest term in x.

- A **proper fraction** is one in which the numerator is a polynomial of *lower* degree than the denominator.
- An **improper fraction** is one in which the numerator is a polynomial of *higher* degree than the denominator.
- When decomposing into partial fractions, the resulting fractions are such that the numerator is exactly ONE degree less than the denominator
- Repeated factors get one fraction per repetition.

1. Improper and proper fractions

Expression	Degree numerator	Degree denominator	Type
$\frac{x^2+5}{x^3}$	2	3	proper
$\frac{x^2+5}{x}$			
$\frac{x^4}{x^3+1}$			
$\frac{x^2-x+3}{x^5+x^3-2x-1}$			

2. Complete the general form of the numerator for the given denominators

- $\frac{A}{x+1}$
- $\frac{\quad}{4-x^2}$
- $\frac{\quad}{x^3}$
- $\frac{\quad}{x^2+1}$
- $\frac{\quad}{x}$
- $\frac{\quad}{x^2-1}$

3. Write the general form of the partial fractions

- $\frac{3x+5}{(x-3)(2x+1)} = \frac{A}{x-3} + \frac{B}{2x+1}$
- $\frac{3x+1}{(x^2+1)(x+2)} = \frac{\quad}{x^2+1} + \frac{\quad}{x+2}$
- $\frac{2}{(x-1)^2(x+2)} =$
- $\frac{4x}{(x^2+x+1)(x-2)} =$

4. Solve the integral

$$\int \frac{x^4 + 2x^2 + 1}{x^2 + 3x + 2}\, dx$$