1. Find $\frac{d^2y}{dx^2}$ for the parametric curve given by

$$x = 3\cos t + 1, \qquad y = t^3$$

• We start with the first derivative:



• The second derivative is found as

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}},$$

which means we need to find the derivative with respect to t of the first derivative:

$$\frac{d}{dt}\left(\frac{dy}{dx}\right) = \frac{d}{dt}\left(\boxed{\boxed{\boxed{\boxed{\boxed{}}}}\right) = \boxed{\boxed{\boxed{\boxed{\boxed{}}}}$$

• Finally we divide this by dx/dt to find the second derivative:

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} =$$

2. Find $\frac{d^2y}{dx^2}$ at the point (1, -1) for the parametric curve given by x = 2t + 1, $y = t^3 - 1$



Evaluate at the point



- 3. Consider the parametric curve defined as $(3 t^2, t^2 + 3t)$.
 - (a) Which graphs corresponds to the given curve?



4. Find the slope of the tangent line at the point on the curve where it crosses the positive y-axis.

5. Find the point (x, y) on the curve where the tangent line is horizontal.

6. At the point where t = 1/2 is the graph increasing or decreasing?

7. The figure shows a curve with parametric equations $x = 5\cos\theta$, $y = 4\sin\theta$, $0 \le \theta \le 2\pi$ and the point P in the curve corresponding to $\theta = \pi/4$.



- (a) Find the equation of the tangent line at ${\cal P}$
- (b) Find the area of the shaded region

We first find the slope of the tangent line and the coordinates of the point P.



(a) The equation of the tangent line can be found as

$$y -$$
 = $\left(\left. \frac{dy}{dx} \right|_{\theta = \pi/4} \right) \cdot \left(x -$ $\right) \Rightarrow y =$

(b) To find the area we identify two subareas: A_1 is the area enclosed by the curve for $0 \le \theta \le pi/4$, while A_2 is the area of the triangle formed by the tangent line and the coordinate axes. Finally the shaded area is given by





• To find A_1 we solve the integral

$$\int_{\Box} \underbrace{y(\theta) x'(\theta) d\theta}_{\Box} =$$

• To find the area of the triangle, A_T , we need to find the point R where the tangent line crosses the x-axis and use the equation for the area of a triangle.

– Finding R

- Area of a triangle is base times height divided by two.

Finally the area of the shaded region is