1. Find $\frac{d^{2} y}{d x^{2}}$ for the parametric curve given by $\quad x=3 \cos t+1, \quad y=t^{3}$

- We start with the first derivative:

$$
\frac{d y}{d x}=\frac{\frac{d y}{d t}}{\frac{d x}{d t}}=\square
$$

- The second derivative is found as

$$
\frac{d^{2} y}{d x^{2}}=\frac{\frac{d}{d t}\left(\frac{d y}{d x}\right)}{\frac{d x}{d t}}
$$

which means we need to find the derivative with respect to $t$ of the first derivative:

$$
\frac{d}{d t}\left(\frac{d y}{d x}\right)=\frac{d}{d t}(\square)=\square
$$

- Finally we divide this by $d x / d t$ to find the second derivative:

$$
\frac{d^{2} y}{d x^{2}}=\frac{\frac{d}{d t}\left(\frac{d y}{d x}\right)}{\frac{d x}{d t}}=
$$

2. Find $\frac{d^{2} y}{d x^{2}}$ at the point $(1,-1)$ for the parametric curve given by $\quad x=2 t+1, \quad y=t^{3}-1$

$$
\frac{d y}{d x}=\frac{\frac{d y}{d t}}{\frac{d x}{d t}}=\square
$$

$$
\frac{d^{2} y}{d x^{2}}=\frac{\frac{d}{d t}\left(\frac{d y}{d x}\right)}{\frac{d x}{d t}}=
$$

Evaluate at the point

3. Consider the parametric curve defined as $\left(3-t^{2}, t^{2}+3 t\right)$.
(a) Which graphs corresponds to the given curve?

4. Find the slope of the tangent line at the point on the curve where it crosses the positive $y$-axis.
5. Find the point $(x, y)$ on the curve where the tangent line is horizontal.
6. At the point where $t=1 / 2$ is the graph increasing or decreasing?
7. The figure shows a curve with parametric equations $x=$ $5 \cos \theta, y=4 \sin \theta, 0 \leq \theta \leq 2 \pi$ and the point $P$ in the curve corresponding to $\theta=\pi / 4$.
(a) Find the equation of the tangent line at $P$
(b) Find the area of the shaded region


We first find the slope of the tangent line and the coordinates of the point $P$.
$\frac{d y}{d x}=\frac{\frac{d y}{d \theta}}{\frac{d x}{d \theta}}=\frac{\square}{\square=}=$

$$
\left.\frac{d y}{d x}\right|_{\theta=\pi / 4}=\square
$$

- The coordinates of $P$ are $(\square, \square)$
(a) The equation of the tangent line can be found as
$y-\square=\left(\left.\frac{d y}{d x}\right|_{\theta=\pi / 4}\right) \cdot(x-\square) \Rightarrow y=$
(b) To find the area we identify two subareas: $A_{1}$ is the area enclosed by the curve for $0 \leq \theta \leq p i / 4$, while $A_{2}$ is the area of the triangle formed by the tangent line and the coordinate axes. Finally the shaded area is given by

$$
A_{2}=A_{T} \square A_{1} .
$$

- To find $A_{1}$ we solve the integral

$$
\int \square y(\theta) x^{\prime}(\theta) d \theta=
$$

- To find the area of the triangle, $A_{T}$, we need to find the point $R$ where the tangent line crosses the $x$-axis and use the equation for the area of a triangle.
- Finding $R$
- Area of a triangle is base times height divided by two.

Finally the area of the shaded region is

