

**Palmetto Joint Arithmetic, Modularity, and Analysis Series**

**Saturday-Sunday, December 5-6, 2020**

**Titles and Abstracts**

**Francesca Bianchi**, University of Groningen

**Title:** *p-adic heights and p-adic sigma functions on Jacobians of genus 2 curves*

**Abstract:** Extending work of Mazur and Tate on elliptic curves, Blakestad recently constructed a p-adic analogue of the complex sigma function on Jacobians of genus 2 curves. We use Blakestad's function to define, compute and study p-adic heights on such Jacobians. P-adic heights are of much arithmetic interest. Just to cite one application, they figure prominently in the quadratic Chabauty method for the computation of integral and rational points on curves. The talk is partly based on joint work with Enis Kaya and Steffen Müller.

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**Gilyoung Cheong**, University of Michigan

**Title:** *Cohen-Lenstra distributions given by p-adic matrices*

**Abstract:** As heuristics to conjectures about the distribution of the p-part of a random imaginary quadratic field, Friedman and Washington gave two results about the distribution the cokernel of a random integral p-adic square matrices with respect to the Haar measure when the size of the matrix goes to infinity. In this talk, I will give a generalization to these two results and a conjecture. This is joint work with Y. Huang.

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**Alexander Clifton**, Emory University

**Title:** *An exponential bound for exponential diffsequences*

**Abstract:** A 1927 theorem of van der Waerden states that for any positive integer  $r$ , if you partition  $\mathbf{N}$  into  $r$  disjoint subsets, then one of them will contain arbitrarily long arithmetic progressions. This motivates the definition of the van der Waerden number  $W(r,k)$  which is the smallest  $n$  such that partitioning

$\{1, 2, \dots, n\}$  into  $r$  subsets guarantees the presence of an arithmetic progression of length  $k$  contained entirely within the same subset.

It is natural to ask what other arithmetic structures are preserved when partitioning  $\mathbb{N}$  into a finite number of disjoint sets and to define analogues of the van der Waerden numbers for those. One notion to consider is that of a  $D$ -diffsequence, introduced by Landman and Robertson, which is an increasing sequence  $a_1 < a_2 < \dots < a_k$  in which the consecutive differences  $a_i - a_{i-1}$  all lie in some given set  $D$ . Here, we consider the case where  $D$  consists of all powers of  $2$  and define  $f(k)$  to be the smallest  $n$  such that partitioning  $\{1, 2, \dots, n\}$  into  $2$  subsets guarantees the presence of a  $D$ -diffsequence of length  $k$  contained entirely within one subset. In this talk, we will establish that  $f(k)$  grows exponentially and time permitting, we will discuss other choices of  $D$  which behave differently.

I would like my talk to be recorded and added to the channel. I'm not available from 1:15 to 4:15 on Saturday.

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**Pengyong Ding**, Pennsylvania State University

**Title: On a variance associated with the distribution of  $r_3(n)$  in arithmetic progressions**

**Abstract:** There is a long history in studying the asymptotic formula for the variance

$$V(x, Q) = \sum_{q \leq Q} \sum_{a=1}^q |A(x; q, a) - f(q, a)M(x)|^2, \quad \text{where} \quad A(x; q, a) = \sum_{\substack{n \leq x \\ n \equiv a \pmod{q}}} a_n,$$

and  $f(q, a)$  and  $M(x)$  approximately reflect the local and global properties of the real sequence  $\{a_n\}$  respectively. In this talk, we will discuss the corresponding expression when  $a_n = r_3(n)$ , the number of ordered representations of  $n$  as the sum of three positive cubes. The Hardy-Littlewood Circle Method will be used to obtain the result, and some special examples will be discussed at the end of the talk.

I would like my talk to be recorded and added to the channel, and I prefer to talk after 12 pm (either Saturday or Sunday).

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**Elisa Lorenzo García**, Université de Neuchâtel

**Title: Primes of bad reduction for CM curves of genus 3 and their exponents on the discriminant**

(joint work with S. Ionica, P. Kilicer, K. Lauter, A. Manzateanu and C. Vincent)

**Abstract:** Let  $O$  be an order in a sextic CM field. In order to construct genus 3 curves whose Jacobian has CM by  $O$  we need to construct class polynomials, and for doing this we need to control the primes in the discriminant of the curves and their exponents. In previous works I studied the so-called "embedding problem" in order to bound the primes of bad reduction. In the present one we give an algorithm to explicitly compute them and we bound the exponent of those primes in the discriminant for the hyperelliptic case. Several examples will be given.

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**Kevin Gomez**, Vanderbilt University

**Title: Bounds for Coefficients of the  $f(q)$  Mock Theta Function and Applications for Partition Ranks**

**Abstract:** Ramanujan's mock theta function  $f(q)$  is a  $q$ -hypergeometric series with great relevance in the analysis of partition ranks modulo two. Recent work due to Bruinier and Schwagenscheidt expresses the Fourier coefficients of  $f(q)$  as finite algebraic formulas. I discuss the techniques used to obtain effective bounds for the Fourier coefficients from these formulas and the application of these bounds to resolve a conjecture of Hou and Jagadeesan on the convexity of the modulo two rank counting functions. This is joint work with Eric Zhu.

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**Jon Grantham**, IDA/CCS

**Title: Extremely Pointless Curves**

**Abstract:** Recall that the gonality of a curve over a finite field  $k$  is the minimum degree of a  $k$ -morphism to the projective line. For a smooth projective curve of genus  $g$  over a finite field, the gonality cannot exceed  $g+1$ . Equality is only possible for so-called “pointless” curves, though this is not sufficient in general. We call a curve with gonality  $g+1$  an “extremely pointless” curve. We determine all pairs  $(g, q)$  for which there exists an extremely pointless curve of genus  $g$  over the field with  $q$  elements, with three possible exceptions. This is joint work with Xander Faber.

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I prefer Sunday before 1:30.

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**Hester Graves, IDA/CCS**

**Title: "The abc conjecture implies that only finitely many Cullen numbers are repunits"**

**Abstract:** Assuming the abc conjecture with  $\varepsilon = 1$ , we use elementary methods to show that for any integer  $s \geq 2$ , there are only finitely many  $s$ -Cullen numbers that are repunits. More precisely, for fixed  $s$ , there are only finitely many positive integers  $n, b$ , and  $q$  with  $n, b \geq 2$  and  $q \geq 3$  such that  $C_{s,n} = ns^n + 1 = (b^q - 1)/(b - 1)$ . This is joint work with Jon Grantham.

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**Christopher Keyes, Emory University**

**Title: An upper bound for the number of arithmetical structures on a graph**

**Abstract:** Let  $G$  be a connected undirected graph on  $n$  vertices with no loops but possibly multiedges. Given an arithmetical structure  $(r, d)$  on  $G$ , we describe a construction which associates to it a graph  $G'$  on  $n-1$  vertices and an arithmetical structure  $(r', d')$  on  $G'$ . By iterating this construction, we derive an upper bound for the number of arithmetical structures on  $G$  depending only on the number of vertices and edges of  $G$ . In the specific case of complete graphs, possibly with multiple edges, we refine and compare our upper bounds to those arising from counting unit fraction representations. This is joint work with Tomer Reiter.

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**Cezar Lupu**, Texas Tech University

**Title: A Zagier-type formula for the special multiple Hurwitz zeta values.**

**Abstract:** In this talk, we provide a Zagier-type formula for the multiple t-values (special Hurwitz zeta values).

Our formula is similar to Zagier's formulas for multiple zeta values  $\zeta(2, \dots, 2, 3)$  and will involve  $\mathbb{Q}$ -linear combinations of powers of  $\pi$  and odd zeta values. The derivation of the formula for  $t(2, \dots, 2, 3)$  relies on a rational zeta series approach via a Gauss hypergeometric function argument.

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**Josh Males**, University of Cologne

**Title: Traces of cycle integrals of meromorphic modular forms and harmonic Maass forms**

**Abstract:** Kohnen and Zagier studied the traces of cycle integrals of certain cusp forms, in particular showing that simple linear combinations of them are rational. A direct generalisation of the function studied yields meromorphic modular forms, which we concentrate on. We will see how to relate the traces of cycle integrals to a locally harmonic Maass form and in turn a certain Siegel theta lift. Borrowing ideas of a recent breakthrough paper of Bruinier-Ehlen-Yang, we see how to obtain the theta lift as the constant term in a  $q$ -series involving coefficients of harmonic Maass forms and theta functions. Some examples involve Hurwitz class numbers, and Andrews'  $spt$  function. As a by-product, we obtain an elegant proof of rationality. This is joint work with C. Alfes-Neumann, K. Bringmann, and M. Schwagenscheidt.

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I am in GMT+1, so preferably not too late on either day.

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**Vlad Matei**, University of Tel Aviv

**Title: Counting pairs of projective plane curves with a prescribed number of  $F_q$  intersection points**

**Abstract:** In joint work with Nathan Kaplan we study the question of counting pairs of projective plane curves, one of degree  $d$  and one of degree  $e$ , that intersect in  $k$   $\mathbb{F}_q$  points where  $0 \leq k \leq de$ . The main term for this count was obtained by Entin as a corollary to the computation of the monodromy group. We obtain explicit formulas for  $(d,e)=(2,2), (2,3), (3,3)$  and in ongoing work we obtain explicit formulas for the general case  $(d,e)$  where  $d=2,3$  and  $e$  is any degree.

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**Mentzelos Melistas**, University of Georgia

**Title: A divisibility related to the BSD Conjecture**

**Abstract:** The Birch and Swinnerton-Dyer (BSD) conjecture asserts that the size of the group of rational points of an elliptic curve, as well as several other invariants, are related to the behavior of an associated analytic object, the L-function of the curve. After discussing the BSD conjecture for elliptic curves over the rationals, we will focus on the analytic rank zero case and discuss a conjecture of Agashe, which is a consequence of the BSD. Finally, we will present a theorem that proves Agashe's conjecture.

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**Eshita Mazumdar**, ISI Bangalore, India

**Title: Extremal Problem related to Davenport Constant**

**Abstract:** Typically an extremal problem deals with the problem of estimating the maximum or minimum possible cardinality of a collection of finite objects that satisfies certain requirements. In my talk I am going to present my most recent research works related to extremal problems. For a finite abelian group  $G$  and  $A$  subset of  $[1, \exp(G) - 1]$ ; the  $A$ -weighted Davenport Constant  $DA(G)$  is defined to be the least positive integer  $k$  such that any sequence  $S$  with length  $k$  over  $G$  has a non-empty  $A$ -weighted zero-sum subsequence. The original motivation for introducing Davenport Constant was to study the problem of non-unique factorization domain over number fields. The precise value of this invariant for any group and for any set  $A$  is still unknown. I will present an Extremal Problem related to Weighted Davenport Constant, which we introduced and discuss several exciting results for any finite abelian group. It is a joint work with Prof. Niranjana Balachandran.

I would prefer to speak between 9:00am - 10:30am (IST) on Sunday.

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**Rachel Newton**, University of Reading

**Title: Evaluating the wild Brauer group**

**Abstract:** The local-global approach to the study of rational points on varieties over number fields begins by embedding the set of rational points on a variety  $X$  into the set of its adelic points. The Brauer-Manin pairing cuts out a subset of the adelic points, called the Brauer-Manin set, that contains the rational points. If the set of adelic points is non-empty but the Brauer-Manin set is empty then we say there's a Brauer-Manin obstruction to the existence of rational points on  $X$ . Computing the Brauer-Manin pairing involves evaluating elements of the Brauer group of  $X$  at local points. If an element of the Brauer group has order coprime to  $p$ , then its evaluation at a  $p$ -adic point factors via reduction of the point modulo  $p$ . For  $p$ -torsion elements this is no longer the case: in order to compute the evaluation map one must know the point to a higher  $p$ -adic precision. Classifying  $p$ -torsion Brauer group elements according to the precision required to evaluate them at  $p$ -adic points gives a filtration which we describe using work of Bloch and Kato. Applications of our work include addressing Swinnerton-Dyer's question about which places can play a role in the Brauer-Manin obstruction. This is joint work with Martin Bright.

I would like my talk to be recorded.

**Sudhir Pujahari**, The University of Hong Kong

**Title:** Distribution of moments of trace of Frobenius in arithmetic progressions.

**Abstract:** In this talk we will study moments of the trace of Frobenius of elliptic curves if the trace is restricted to a fixed arithmetic progression. In particular, we fix the arithmetic progression and consider the ratio of the  $2k$ -th moment to the zeroth moment as one varies the size of the finite field  $F_{p^r}$ . We will see asymptotic formulas for these ratios for cases for which the prime  $p$  goes to infinity for fixed  $r$  and cases where the power  $r$  goes to infinity with fixed  $p$ . Moreover, we will see that these results follow from similar asymptotic formulas relating sums and moments of Hurwitz class numbers where the sums are restricted to certain arithmetic progressions. This is a joint work with Kathrin Bringmann and Ben Kane.

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**Anwesh Ray**, University of British Columbia

**Title:** Euler characteristics in Iwasawa theory and their congruences

**Abstract:** Let  $p$  be a prime number. Two elliptic curves are congruent modulo  $p$  if they are  $p$ -adically close to each other from a Galois theoretic perspective. The generalized Euler characteristic is an intrinsic arithmetic invariant associated to an elliptic curve which is of interest from the perspective of the  $p$ -adic Birch and Swinnerton Dyer conjecture. It is shown that if two elliptic curves are congruent modulo  $p$ , then there is an explicit relationship between the Euler characteristics. This is joint work with R.Sujatha.

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**Arvind Suresh**, University of Georgia

**Title: New points, rank growth, and the twisted Prouhet-Tarry-Escott problem**

**Abstract:** For a variety  $X/K$  and finite extension  $L/K$ , an  $L$ -rational point  $P$  of  $X$  is a *new point* if it is not  $F$ -rational for any proper subfield  $F$  of  $L$ . Given an extension  $L/K$  and positive genus  $g$  (small relative to  $[L:K]$ ), it is not known if there exist any genus  $g$  curves with a new point over  $L$ . In this talk, we generalize work of Liu--Lorenzini, Rohrlich, and Matsuno to give some constructions of hyperelliptic curves with new points over specified field extensions  $L/K$ , and whose Jacobians' Mordell-Weil ranks grow at  $L$ . Some of these theorems rely on finding solutions to a "twisted" version of the Prouhet-Tarry-Escott problem.

[Speak on Sunday] I would like my talk to be recorded and added to the channel.

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**Zack Tripp**, Vanderbilt University

**Title: Log-concavity of the  $k$ -colored partition functions**

**Abstract:** Recently, a number of papers have considered mixtures between additive and multiplicative behavior in inequalities for partition functions. In particular, Chern--Fu--Tang and Heim--Neuhauser gave conjectures on inequalities for coefficients of powers of the partition generating function. These conjectures were posed in the context of colored partitions and the Nekrasov--Okounkov formula. In this talk, we will discuss partial progress towards the Heim--Neuhauser conjecture, along with the solution to the Chern--Fu--Tang conjecture. In particular, this will imply that the  $k$ -colored partition function is log-concave for integral  $k$  bigger than 2. This is based on joint work with Kathrin Bringmann, Ben Kane, and Larry Rolen.

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**Shaoyun Yi**, University of South Carolina

**Title: Analogues of Alladi's formula over global function fields**

**Abstract:** In this work, we show an analogue of Kural et al.'s result on Alladi's formula for global function fields. As applications, we get the analogue of Dawsey's and Sweeting and Woo's results to the Chebotarev Density Theorem for function fields, and the analogue of Alladi's result to the Prime Polynomial Theorem for arithmetic progressions. We also display a connection between the Mobius function and the arithmetical property of elliptic curves over number fields. This is joint work with Lian Duan and Biao Wang.

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**Daozhou Zhu**, Clemson University

**Title:** *TBA*

**Abstract:** *TBA*