

Algebraic number theory (Spring 2013), Homework 5a

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Due Friday, March 22

(These exercises concern the Tuesday lectures on valuations.)

1. (5 points) Prove, without quoting any theorems about valuations, that $|z|$ is the only valuation of \mathbb{C} which extends the usual one on \mathbb{R} .
2. (20 points) Do Exercises 1 and 2 from Neukirch, p. 152. Feel free to assume that the ‘henselian field’ is a finite extension of \mathbb{Q}_p and ignore any separability hypotheses (which are automatic in this case).

Conclude that for any given p and n , there are only finitely many extensions of \mathbb{Q}_p of degree n . (You may find it interesting to look at the Jones-Roberts database for a list of such extensions.)

3. (5+ points) Exhibit cubic extensions K and L of \mathbb{Q}_5 which are ramified and unramified, respectively. (Optional: generalize.)
4. (10 points) Exhibit two different cubic ramified extensions of \mathbb{Q}_3 , referring to the Jones-Roberts database if you like. (The work is in proving that the fields are not isomorphic.)