Homework 11, Math 702 – Frank Thorne (thorne@math.sc.edu)

You are welcome and encouraged to collaborate, but please write up your own solutions.
Due Friday, April 20, 2018.

1. Determine the Galois closure of the field $\mathbb{Q}(\sqrt{1+\sqrt{2}})$ over $\mathbb{Q}$.

2. Let p be prime. The field extension $\mathbb{F}_p(x,y)/\mathbb{F}_p(x^p,y^p)$ doesn't satisfy the hypotheses of the primitive element theorem. Compute its degree (and in particular prove that it is finite) and exhibit an infinite number of intermediate subfields.

3. Determine a rational polynomial which has $\cos(40^\circ)$ as a root.

4. Determine the Galois group of $x^4+8x+12$.

5. Let θ be a root of x^3-3x+1. Prove that $\mathbb{Q}(\theta)$ is Galois and cyclic over $\mathbb{Q}$, so that x^3-3x+1 splits over $\mathbb{Q}(\theta)$. Compute the other two roots of this polynomial, as polynomials in θ.