# Homework 4, Math 701 – Frank Thorne (thorne@math.sc.edu)

**Instructions:** You are welcome and encouraged to collaborate, but please write up your own solutions.

Due Monday, October 30, 2017.

1. Exhibit two distinct 2-Sylow subgroups of $S_5$ and an element of $S_5$ that conjugates one to the other.

2. Consider the group $\mathrm{SL}_2(\mathbb{Z}/3) = \mathrm{SL}_2(\mathbb{F}_3)$ of $2 \times 2$ matrices with entries in $\mathbb{Z}/3$ (i.e., in the finite field $\mathbb{F}_3$) whose determinant is 1.

   (Note that the determinant involves multiplying as well as subtracting elements of $\mathbb{Z}/3$. If you are not familiar with finite fields, just note that both multiplication and addition are carried out modulo 3.)

   (a) Prove that this group has 24 elements in it.

   (b) Describe all of its 2-Sylow and 3-Sylow subgroups.

   (c) Prove that the center $Z$ of $\mathrm{SL}_2(\mathbb{Z}/3)$ consists of $\pm I$, and that $\mathrm{SL}_2(\mathbb{Z}/3)/Z \simeq A_4$.

3. Construct a non-abelian group of order 75.

4. Classify all groups of order 28 (there are four isomorphism types).