Homework 3 - Math 580, Frank Thorne (thornef@mailbox.sc.edu)

Due Friday, September 25

(1) Dudley, p. 32-33, 5, 8, 10, 11, 15, 18.

(2) Dudley, p. 40-41, 1, 3, 6, 7, 12, 20.

(3) In class, I drew a square grid, where the rows were residue classes $\pmod 3$ and the columns were residue classes $\pmod 5$, and demonstrated that every square of the grid could be filled by some integer.

Repeat this, with the following pairs of moduli. What do you get? Look closely at the patterns you see!

(a) 5 and 8

(b) 2 and 6

(c) 8 and 6

(d) 10 and 15

(4) **Bonus problem:** The Chinese Remainder Theorem asserts the following. Suppose that m_i are pairwise coprime integers. Then, the system of congruences

$$x \equiv a_i \pmod{m_i} \quad i = 1, 2, \ldots, k$$

has a unique solution modulo $m_1, m_2, \cdots m_k$.

Suppose that the m_i are not coprime; then the Chinese Remainder Theorem is not true as stated. What is true? Formulate and prove a version of the theorem that holds in this case.