Midterm Examination 1 - Math 580, Frank Thorne (thorne@math.sc.edu)

Due Friday, October 9, 2020 at 1:00

Instructions: This is a **take-home, two hour timed exam.** Please set aside any two-hour time period of your choice, and take the exam in one sitting.

Please work without books, notes, calculators, internet, or any assistance from others. Be sure to show all your work and explain what you are doing!

GOOD LUCK!

– Questions 1-7 are worth 12 points each; 16 points + 5 bonus for Question 8 –

(1) Find all the integer solutions to (a) $15x + 21y = 51$, and (b) $15x + 21y = 52$.

(2) Solve each of the following congruences:

(a) $11x \equiv 22 \pmod{31}$,

(b) $11x \equiv 22 \pmod{33}$,

(c) $11x \equiv 21 \pmod{33}$.

Note: these are to be solved individually, not simultaneously.

(3) Determine the least residue of $7^{2020} \pmod{11}$.

(4) Solve the system of congruences $x \equiv 4 \pmod{9}$ and $x \equiv 7 \pmod{11}$.

(5) Prove or disprove: $(k, n + k) = d$ if and only if $(k, n) = d$.

(6) Let p be an odd prime. Then by Wilson's theorem you know that $(p-1)! \equiv -1 \pmod{p}$. What is $(p-2)! \pmod{p}$? Prove your claim.

(7) Suppose that $(a, m) = 1$. Prove that the integers

$$a, 2a, 3a, \ldots, (m-1)a$$

all represent different residue classes $\pmod{m}$.

(8) Suppose you write out a 24×30 table as follows. The rows correspond to the residue classes 0 (mod 24) through 23 (mod 24), and the columns correspond to the residue classes 0 (mod 30) through 29 (mod 30).

For each x from 0 to 719, you write x in the box given by $x \pmod{24}$ and $x \pmod{30}$. Note that multiple values of x can go in the same box. For example, 171 and 291 would go in the same box because $171 \equiv 291 \equiv 3 \pmod{24}$ and $171 \equiv 291 \equiv 21 \pmod{30}$.

(a) Find a box in the table which will be empty.

(b) Determine all the numbers which will appear in the same box as 0.

(c) Determine all the numbers which will appear in the same box as 171 and 291.

(d) Determine, without proof, how many boxes have numbers in them, and how many numbers each such box has.

(e) (**Bonus**) Prove your claim in the previous part.