Optional Midterm Exam 2 – Vector Calculus, Frank Thorne (thorne@math.sc.edu)

Due Monday, April 20, 2020 at 9:40 a.m.

This exam is **optional**. If it helps your grade, it will be included. If it hurts your grade, or if you don't turn it in, then it will be excluded and your other scores will be scaled up (multiplied by $\frac{100}{85}$ so as to total 100%).

Instructions and Advice:

- This is a 2 hour take home exam, covering the material of Chapters 5.5 and 6. Please take it in one continuous 2 hour block of your choosing. (It is recommended that you find somewhere quiet where you can work without interruption.)
- You are allowed to refer to one 4x6 index card during the exam; this may be prepared with anything you like, front and back – as long as it's in your own handwriting and you prepare it yourself.

  Other than that, no books, notes, calculators, cell phones, or assistance from others.
- **Draw pictures, and write complete sentences, where appropriate. Be clear, write neatly, explain what you are doing, and show your work.**
- **Second Chance Opportunity.** After you finish the exam, you are encouraged to use your book, notes, the Internet, etc. to check your work.

  If you find any mistakes, then you may submit a list of corrections and/or new solutions. Please distinguish these from the exam itself (i.e., turn in your exam solutions even if you discover they are wrong). You will get up a bonus of to a quarter of the points you lost.
- The completed exam and Second Chance are due by email (`thorne@math.sc.edu`) to me by 9:40 a.m. on Monday, April 20.
- Once you have taken the exam, please don't talk to anyone (except me) about it before 9:40 a.m. on April 20.

GOOD LUCK!

20 points each question.

(1) Determine the value of

$$\int\int_D \sqrt{\frac{x+y}{x-2y}}dA,$$

where D is the region in $\mathbb{R}^2$ enclosed by the lines $y = x/2$, $y = 0$, and $x + y = 1$.

(2) Let C be a level set of a function $f(x, y)$; i.e., a set of the form

$$\{(x, y) \in \mathbb{R}^2 \ : f(x, y) = c\}$$

for some fixed c and some scalar-valued smooth function $f : \mathbb{R}^2 \mapsto \mathbb{R}$.

(a) If the boundary of C is a simple, closed loop, then show that $\int_C \nabla f \cdot d\vec{s} = 0$.

(b) Draw a picture and explain your conclusion geometrically.

(3) A *hypocycloid* is given by the equation

$$\vec{x}(t) = (a\cos^3 t, a\sin^3 t), \quad 0 \le t \le 2\pi$$

for a constant a. Sketch the hypocycloid, and use Green's theorem to find its area.

(4) Consider the vector field $\vec{F}$ in two dimensions, given by

$$\vec{F} = \frac{x + xy^2}{y^2}\vec{i} - \frac{x^2+1}{y^3}\vec{j}.$$

(i) Determine whether $\vec{F}$ is conservative.

(ii) Determine a scalar potential for $\vec{F}$.

(iii) Find the work done by $\vec{F}$ in moving a particle along the parabolic curve $y = 1 + x - x^2$ from $(0, 1)$ to $(1, 1)$.

(5) Consider the vector field $\vec{F}$ in two dimensions, given by

$$\vec{F} = \frac{y}{x^2+y^2}\vec{i}.$$

(i) Evaluate $\oint_C \vec{F} \cdot d\vec{s}$, where C is the unit circle $x^2 + y^2 = 1$, traversed in the counterclockwise direction.

(ii) Can the line integral above be evaluated using Green's theorem? Why or why not?