Homework 0 - Math 550H, Frank Thorne (thorne@math.sc.edu)

Due Friday, January 24, 2020

Special Instructions: Please treat this like an exam. Take two hours exactly, and don't use any books, notes, or other tools.

You might not know how to answer all the questions. You might not know what some of them even mean. That is okay – the purpose is to introduce some of the ideas that will appear in the course, and to get a sense of where everyone is.

You will earn full credit if you make a good faith effort to answer each of the questions. If you don't figure out the solution, or you don't understand what the question means, then guess, explain what do you do know, what you're confused about, address the question somehow.

(1) What is the average value of the function $f(x,y) = x^2 + y^2$ over the circle

$$\{(x,y) \in \mathbb{R}^2 \ : \ x^2 + y^2 \leq 1\}?$$

(2) An object travels in a circle at a constant speed. Prove that the object's acceleration is always directed towards the center of the circle.

(3) Define the *definite integral* $\int_a^b f(x)dx$. Draw a picture illustrating your definition, and explain why it gives the area underneath the curve.

In addition, what does the expression dx mean above?

(4) A function $f : \mathbb{R}^2 \mapsto \mathbb{R}^2$ is given by

$$f(x,y) = (ax + by, cx + dy),$$

where a, b, c, and d are all real numbers.

(a) Determine conditions on a, b, c, and d which guarantee that f is onto (i.e., that its image is all of $\mathbb{R}^2$).

(b) Determine conditions on a, b, c, and d which guarantee that if $f(x,y) = (0,0)$, then $(x,y) = (0,0)$.

(c) What is the relationship between your two answers above? Explain as best you can.

(5) Let C be the circle $\{(x,y) \in \mathbb{R}^2 \ : \ x^2 + y^2 = 1\}$.

(a) Consider the line integral

$$\int_C (-ydx + xdy),$$

where C is oriented in the counterclockwise direction.

What does this line integral mean? (If you don't know, then guess.)

(b) Evaluate (or guess) the value of the line integral.

(c) Repeat with the integral

$$\int_C (xdx + ydy).$$