Homework 2 - Math 547/702I, Frank Thorne (thornef@mailbox.sc.edu)

Due Monday, February 2, 2015

Reading: Please read Chapter 17 of Saracino.

(1) **You are strongly encouraged to look at the remainder of the exercises in Chapter 16 and attempt some of them.**

(2) 17.1, 17.2 (fairly brief answers are totally fine).

(3) 17.6, 17.7, 17.9

(4) 17.12. (This is very easy, but potentially confusing. What is the difference between the conditions stated and the definition of an ideal?)

(5) 17.13, 17.14, 17.15.

(6) 17.16, 17.17, 17.18 (These are some genuine red meat, not just definition pushing!)

(7) 17.33, 17.34, 17.35.

(8) **Note: the rest of the exercises in Chapter 17 are really great too! I recommend doing all of them!**

(9) **Bonus. Baby Math 788.** Consider the ring $\mathbb{Z}[i] = \{a + bi \ : \ a, b \in \mathbb{Z}\}$. Say as much as you can about this ring. For example:

  (a) What are the units?

  (b) What are all of the ideals? (In particular: are there any which are not principal?)

  (c) Write down some examples of some ideals which are or are not prime and/or maximal.

  (d) What does $\mathbb{Z}[i]/I$ look like for some different ideals I?

  (e) Two particularly interesting choices are the principal ideals (3) and $(2 + i)$. What do $\mathbb{Z}[i]/(3)$ and $\mathbb{Z}[i]/(2+i)$ look like? Are they integral domains? Are they fields? How many elements do they have? Does either look familiar to you?

If you get stuck on one part, keep going and try the next.