Homework 1 - Math 546H, Frank Thorne (thornef@mailbox.sc.edu)

Due Thursday, September 11

Please read Saracino through Chapter 3, although you can skip the part about $\mathbb{Z}_n$ for now.

(1) Saracino, Chapter 2: 2.7, 2.12.

(2) Saracino, Chapter 3: 3.3, 3.4, 3.5, 3.6, 3.7, 3.9, 3.10-3.13.

(3) Consider the following groups:

(a) G_1, the group of bijective (i.e., one-to-one and onto) functions from the set $\{1, 2, 3\}$ to itself.

(b) G_2, the group of strings of letters a and b, where the group operation is concatenation, and where the identity e is equal to the identity string, subject to the simplification rules

$$aaa = bb = e, \quad aab = ba.$$

(c) G_3, the group of the following 2×2 matrices:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}, \begin{pmatrix} \frac{1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{-\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}, \begin{pmatrix} -\frac{1}{2} & \frac{-\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}, \begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}.$$

(d) G_4, the group given by the following multiplication table:

	e	a	b	c	d	f
e	e	a	b	c	d	f
a	a	b	e	f	c	d
b	b	e	a	d	c	f
c	c	d	f	e	a	b
d	d	f	c	b	e	a
f	f	c	d	a	b	e

(i) How many elements does each of these groups have?

(ii) Explain how you would prove in detail that each of the above in fact is a group. Whenever there are many tedious details to be verified, give a single example of a typical verification.

(iii) Prove that none of the above groups is abelian.

(iv) For each of these groups, find elements x, y, and z such that $x^2 \neq e$ and $(xy)^2 \neq x^2y^2$. (Recall exercises 3.11 and 3.12 of the book.)

(v) **Bonus. REALLY IMPORTANT AND COOL!!!!** All of these groups are in fact 'the same'. Try to make precise what this means. You can say more than just that they have the same number of elements and none of them are abelian!

Now, prove that the groups G_1 and G_3 are the same in the sense you just defined. But do not work with messy equations involving matrices! Instead, draw a picture of what the matrices represent, and then try to force the numbers $\{1, 2, 3\}$ into your picture.