

## Homework 0 - Math 531, Frank Thorne (thornef@mailbox.sc.edu)

Due Wednesday, August 29

This homework is **optional**. If you do hand it in, it will be graded *extremely* leniently (which the rest of the homeworks won't be).

This homework involves *writing proofs*. This is an essential skill for Math 531. If you find this homework difficult, please be warned that Math 531 might be an uphill climb, and seek help from me and others if you need it.

With all proofs, please explain yourself clearly and in complete English sentences.

- (1) Prove that a triangle cannot have two obtuse angles. (*Obtuse* means greater than 90 degrees.)
- (2) Prove that the sum of two even integers is even, that the sum of two odd integers is even, and that the sum of an even and an odd integer is odd.
- (3) Prove that  $1 + 3 + 5 + \cdots + (2n - 1) = n^2$ . Here is one idea for a nice proof: Draw a big square and divide it into  $n$  rows and  $n$  columns (i.e. into  $n^2$  small squares). There is one square in the top left, there are three touching it, and so on.
- (4) Prove that the area of a unit circle is somewhere between 1 and 4. (The answer is  $\pi$ , but you are not allowed to quote this fact without proving it.)
- (5) You draw three lines through a circle, so that each line intersects the circle twice. In how many regions might this divide the circle? Determine the complete list of possibilities, with proof.
- (6) Prove that there is no positive rational number (i.e., fraction) which is smaller than all other positive rational numbers.