Final Examination - Math 142, Frank Thorne (thorne@math.sc.edu)

Thursday, December 12, 2013

Instructions and Advice:

- There are fifteen questions, some of which are shorter than others.
- You are welcome to as much scratch paper as you need. Turn in everything you want graded, and throw away everything you do not want graded.
- **Draw pictures where appropriate.** If you have any doubt, then a picture is appropriate.
- Be clear, write neatly, explain what you are doing, and show your work. **This is especially important for earning partial credit** in case your work contains one or more mistakes. Be warned that **work I cannot understand will not receive any credit.**
- 150 minutes is a long time. Don't dilly-dally, but don't rush. **You are strongly advised to take the entire 150 minutes to complete the examination.** If you finish early, you have the opportunity to check your work.
- You are welcome and encouraged to refer to the list of convergence tests provided with the exam.
- Please work without books, notes, calculators, or any assistance from others.
- I will be at the front of the room; if you have any questions, feel free to ask me.

GOOD LUCK!

(1) Evaluate

$$\int_0^1 xe^{-x^2}dx.$$

(2) Evaluate

$$\int_0^1 \frac{x-1}{x^2+3x+2}dx.$$

(3) Evaluate

$$\int \frac{x^3}{\sqrt{x^2+100}}dx.$$

(4) What is an *improper integral*? What makes it improper? Give at least two examples that describe different types of improperness.

(5) Sketch the region enclosed by the curves listed below. Decide whether to integrate with respect to x or y. Draw a typical approximating rectangle and label its height and width. Then, find the area of the region.

$$y = x^2,\ y^2 = x.$$

(6) Find the area of a square pyramid with base length b and height b.

(7) Do 10.1, 27 from Stewart.

(8) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the curve given. For which values of t is the curve concave upward?

$$x = t + \ln(t),\ y = t - \ln(t)$$

(9) Sketch the curve given by the polar equation $r^2\theta = 1$.

(10) State and derive the formula for the sum of a geometric series.

(11) Use the integral test to determine whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{1}{n^3+n}.$$

If the series diverges, then draw a graph which represents both the series and the integral youre comparing it to.

If the series converges, give upper and lower bounds on the value of your series which are guaranteed to be accurate within 0.01. Draw a graph which represents your lower bound.

(12) Using the comparison test, or otherwise, determine whether the following series converges or diverges:

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}.$$

(13) Using the ratio test, or otherwise, determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=1}^{\infty}(-1)^n\frac{(1.1)^n}{n^4}.$$

(14) Are there any power series which converge for no values of x?

(15) Compute $1/e$, as a fraction, to fairly good accuracy. Your estimate should plausibly be within $\frac{1}{100}$, but you don't need to prove this.