Examination 3 - Math 142, Frank Thorne (thorne@math.sc.edu)

Thursday, November 21, 2013

Instructions and Advice:

- You are welcome to as much scratch paper as you need. Turn in everything you want graded, and throw away everything you do not want graded.
- **Draw pictures where appropriate.** If you have any doubt, then a picture is appropriate.
- Be clear, write neatly, explain what you are doing, and show your work. **This is especially important for earning partial credit** in case your work contains one or more mistakes. Be warned that **work I cannot understand will not receive any credit.**
- 75 minutes is a long time. Don't dilly-dally, but don't rush. **You are strongly advised to take the entire 75 minutes to complete the examination.** If you finish early, you have the opportunity to check your work.
- This exam is accompanied by a list of convergence tests which you should freely refer to. Please work without books, notes, calculators, or any assistance from others.
- I will be at the front of the room; if you have any questions, feel free to ask me.

GOOD LUCK!

(1) (10 points) Does the sequence

$$a_n = \arctan(2n)$$

converge or diverge? If it converges, find the limit. If it diverges, explain why.

(2) (10 points) Does the sequence

$$a_n = \frac{5^{n+3}}{7^n}$$

converge or diverge? If it converges, find the limit. If it diverges, explain why.

(3) (10 points) Does the series

$$3 + 2 + \frac{4}{3} + \frac{8}{9} + \cdots$$

converge or diverge? If it converges, find its sum.

(4) (10 points) The integral test shows that the series

$$\sum_{n=1}^{\infty} \frac{1}{n^2 + 9}$$

is convergent. Explain why.

(14 points) Give an upper and a lower bound for the value of this series accurate within 0.1. Draw and explain a graph which represents your lower bound.

(5) (12 points)

Use the comparison test (or any other test) to determine whether the series converges or diverges. If it converges, determine an explicit upper bound for the value of the series.

$$\sum_{n=1}^{\infty} \frac{n^2 - 1}{n^4 + 1}$$

(6) (12 points) Use the comparison test (or any other test) to determine whether the series converges or diverges. If it converges, determine an explicit upper bound for the value of the series.

$$\sum_{n=1}^{\infty} \frac{2 + \ln(n)}{5\sqrt{n} - 1}$$

(7) (10 pints) Test the alternating series for convergence or divergence. If it converges, determine an estimate for its value which is accurate within 0.25.

$$\sum_{n=2}^{\infty} (-1)^n \frac{n}{\ln(n)}$$

(8) (10 points) Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n} \frac{10^n}{(n+1)4^{2n+1}}$$