Thompson's definition of the derivative is as follows. Suppose two variables x and y depend on each other in some way. Then, if one of the variables changes, the other does too.

Suppose we add some small amount dx to x, causing it to change to $x + dx$. Then, this will cause y to change to a value $y + dy$. The derivative is the ratio $\frac{dy}{dx}$.

In evaluating $\frac{dy}{dx}$ Thompson uses a shortcut described in Chapter 2. The quantities dy and dx are small but non-negligible, but if they are small enough then their squares and higher powers are so small as to be negligible.

For example, if $y = x^2$ and x changes to $x + dx$, then y changes to

$$y + dy = (x + dx)^2 = x^2 + 2x\,dx + (dx)^2$$
$$= x^2 + 2x\,dx = y + 2x\,dx$$

So $$\frac{dy}{dx} = \frac{2x\,dx}{dx} = 2x.$$

Stewart gives the definition (writing $y = f(x)$)

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}.$$

Stewart's Δx is the same as Thompson's dx. The definition is more complicated, and the purpose is to formalize and make rigorous the notion that powers of dy and dx are small enough to ignore.