

Homework 1 - Math 142, Frank Thorne (thornef@mailbox.sc.edu)

Due Friday, August 29

Important: As with everything else in life, being right is not enough. Please show your work, write in complete sentences, and explain your reasoning clearly.

Thompson. Read Chapters 1-3 of Thompson, and in approximately a page (of mostly plain English, but include equations and diagrams where appropriate) answer the following question.

Thompson defines the notion of a *derivative* without explicit reference to a *limit*. What is Thompson's definition, and how does it relate to the usual definition in Stewart?

Required problems.

- (a) What is a function? (This is the most important question in all of mathematics.)
- (b) Suppose that f is a function whose domain and range are subsets of the real numbers. Explain how to draw the graph of f , and what the graph represents.

- (c) Explain what it means to say that

$$\lim_{x \rightarrow a} f(x) = c.$$

- (d) Define the *derivative* of a function. Define it using an equation, and also explain your definition in English. In addition, draw a picture and explain why your equation describes the tangent line to the graph.
- (e) Define an *inverse function*. Explain how to find the derivative of $f^{-1}(x)$, if you know the derivative of $f(x)$.
- (f) Define the functions $\sin(x)$, $\cos(x)$, $\tan(x)$, $\csc(x)$, $\sec(x)$, $\cot(x)$, e^x , and $\ln(x)$, and say what each of their derivatives is. For $\sin(x)$, $\cos(x)$, and e^x , it is okay to just state the derivatives. For the other functions, explain how you got your answer. (You will want to use the derivatives of \sin , cosine, and e^x !)
- (g) State the power rule, the product rule, the quotient rule, and the chain rule for finding derivatives.
- (h) Stewart, Ch. 3 Review, pp. 262-263: 6-13, 57, 89 (do (d) without a graphing calculator), 93.
- (i) Define the *antiderivative* of a function.
- (j) Define the *definite integral*

$$\int_a^b f(x) dx.$$

Give an equation, and explain why your equation gives the area underneath the curve from $x = a$ to $x = b$.

- (k) What does the fundamental theorem of calculus say? Why is it true? Explain thoroughly.

(l) Stewart, Ch. 5.3, 19-24.

Additional problems.

(a) Stewart, Ch. 3 Review, 1-5, 14-22.

(b) Stewart, Ch. 5.3, 25-32.

Bonus (2 points).

(a) What is the Mean Value Theorem, and why does anybody care about it?