

Final Exam (Version B) - Math 142, Frank Thorne (thorne@math.sc.edu)

Tuesday, December 10, 2019

Instructions and Advice:

- There are twelve questions.
- No books, notes, calculators, cell phones, or assistance from others.
- You are welcome to as much scratch paper as you need. Turn in everything you want graded. Whatever you don't want graded, put in a separate pile and I will recycle it.
- **Draw pictures, and write complete sentences, where appropriate. Be clear, write neatly, explain what you are doing, and show your work. If (for example) you claim that a series converges or diverges, then thoroughly explain how you know.**
- You are welcome to use any formulas for power series which you know. State explicitly any formula which you are using.
- If asked to compute a power series, either write in sigma notation or compute through (at least) the x^3 term.
- Feel free to refer to the list of convergence tests provided with this exam.

GOOD LUCK!

- (1) What is the formula for integration by substitution? Why does it work?
- (2) Suppose that f is continuous on $(-\infty, b]$. What does $\int_{-\infty}^b f(x)dx$ mean? What does it mean for it to converge or diverge?
- (3) Find the area of the region enclosed by the curve $y = x \cos x$ and the x -axis (see the accompanying figure) for $3\pi/2 \leq x \leq 5\pi/2$.

- (4) Evaluate

$$\int_0^{1/2\sqrt{2}} \frac{2 dx}{\sqrt{1-4x^2}}.$$

- (5) The integral

$$\int_0^2 \frac{dx}{\sqrt{|x-1|}}$$

converges. Evaluate it.

- (6) Does the sequence $\{a_n\}$, given by

$$a_n = \frac{2n+1}{1-3\sqrt{n}}$$

converge or diverge? If it converges, find its limit.

(7) Does the series

$$\sum_{n=1}^{\infty} \frac{1 + \cos n}{n^2}$$

converge or diverge? Use any method, and give a reason for your answer.

(8) Does the series converge or diverge? Use any method, and give a reason for your answer.

$$\sum_{n=1}^{\infty} \frac{n \ln n}{(-2)^n}$$

(9) Find the series' radius and interval of convergence. For what values of x does the series converge absolutely, and for what values does it converge conditionally?

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{n^3 3^n}.$$

(10) Find the Taylor series generated by f at $x = 1$, where

$$f(x) = 2x^3 + x^2 + 3x - 8.$$

(11) Parametric equations and parameter intervals are given for the motion of a particle in the xy -plane. Identify the particle's path by finding a Cartesian equation for it. Graph the Cartesian equation. Indicate the portion of the graph traced by the particle and the direction of motion.

$$x = 4 \cos t, \quad y = 2 \sin t, \quad 0 \leq t \leq 2\pi.$$

(12) Find the Cartesian coordinates of the following points (given in polar coordinates):

(a) $(\sqrt{2}, \pi/4)$

(b) $(1, 0)$

(c) $(0, \pi/2)$