Final Examination (version 4) - Math 141, Frank Thorne (thorne@math.sc.edu)

December 2015

Please work without books, notes, calculators, or any assistance from others. Please **show all your work, explain yourself clearly, draw pictures where appropriate, and put equals signs where they belong.**

If you have any questions, feel free to ask me. Please do your work on separate paper; you should staple this sheet to your work (put this on top) and turn in everything together.

Each problem is worth 10 points; a total of 160 points is possible. **GOOD LUCK!**

(1) What does the Fundamental Theorem of Calculus say? Explain carefully and thoroughly.

You do not have to explain why it is true. But you must explain both parts.

(2) What is an inflection point? How do you find them? Why are they interesting?

(3) Compute

$$\lim_{x\to\infty} \sqrt{\frac{x^2 - 5x}{x^3 + x - 2}}.$$

(4) Refer to 27 in Figure 1. Match the function with its derivative graphed in one of (a)-(d).

(Explain your reasoning.)

(5) Find the slope of the graph of the function $h(t) = t^3$ at the point $(2, 8)$. Then find an equation for the line tangent to the graph there.

For this problem, use the definition of the derivative at a point, and do not use differentiation rules such as the power, product, or quotient rules.

(6) Find $\frac{dy}{dt}$, if

$$y = (1 + \cot(t/2))^{-2}.$$

(7) Use implicit differentiation to find $\frac{dy}{dx}$ and then $\frac{d^2y}{dx^2}$, if

$$y^2 = e^{x^2} + 2x.$$

(8) The marginal cost of printing a poster when x posters have been printed is

$$\frac{dc}{dx} = \frac{1}{2\sqrt{x}}$$

dollars. Find $c(100) - c(1)$, the cost of printing posters $2 - 100$.

(9) On the morning of a day when the sun will pass directly overhead, the shadow of an 80 ft building on level ground is 60 ft long. At the moment in question, the angle θ the sun makes with the ground is increasing at the rate of 0.27 degrees per minute. At what rate is the shadow decreasing?

(Remember to use radians. The book asked you to express your answer in inches per minute, to the nearest tenth, but you are not required to do so here.)

A diagram is given in Figure 2.

(10) Answer the following questions about the function f whose derivative is given by

$$f'(x) = (x-1)e^{-x}.$$

(a) What are the critical points of f?

(b) On what open intervals is f increasing or decreasing?

(c) At what points, if any, does f assume local maximum and minimum values?

(11) Graph the function

$$y = \frac{\sqrt{1-x^2}}{2x+1}$$

according to the following instructions taken from the book:

Procedure for graphing $y = f(x)$:

(a) Identify the domain of f and any symmetries the curve may have.

(b) Find the derivatives y' and y''.

(c) Find the critical points of f, if any, and identify the function's behavior at each one.

(d) Find where the curve is increasing and where it is decreasing.

(e) Find the points of inflection, if any occur, and determine the concavity of the curve.

(f) Identify any asymptotes that may exist.

(g) Ploy key points, such as the intercepts and the points found in steps (c)-(e), and sketch the curve together with any asymptotes that may exist.

(12) The height above ground of an object moving vertically is given by

$$s = -16t^2 + 96t + 112,$$

with s in feet and t in seconds. Find

(a) The object's velocity when $t = 0$;

(b) The maximum height and when it occurs;

(c) Its velocity when $s = 0$.

(13) Evaluate

$$\int \left(\frac{t^2}{2} + 4t^3\right)\, dt.$$

(14) Evaluate

$$\int x^{1/2} \sin(x^{3/2}+1)\, dx.$$

(15) Find the area of the region graphed in Figure 3.

Clarification: The function is defined piecewise; the labels apply to $t \leq 0$ and $t \geq 0$ respectively.

(16) Find the volume of the solid generated by revolving the region bounded by the following lines and curves about the x-axis:

$$y = x^2, \quad y = 0, \quad x = 2.$$