Final Examination (version 3) - Math 141, Frank Thorne (thorne@math.sc.edu)

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Please work without books, notes, calculators, or any assistance from others. Please **show all your work, explain yourself clearly, draw pictures where appropriate, and put equals signs where they belong.**

If you have any questions, feel free to ask me. Please do your work on separate paper; you should staple this sheet to your work (put this on top) and turn in everything together.

Each problem is worth 10 points; a total of 160 points is possible. **GOOD LUCK!**

(1) What does the Fundamental Theorem of Calculus say? Explain carefully and thoroughly.

You do not have to explain why it is true. But you must explain both parts.

(2) What is the 500th derivative of $f(x) = x^{100}$? Explain why.

(3) Compute

$$\lim_{y \to 0} \frac{5y^3 + 8y^2}{3y^4 - 16y^2}.$$

(4) Say whether the function graphed in Figure 1 is continuous on $[-1, 3]$. If not, where does it fail to be continuous and why?

(5) Find the slope of the curve $y = 5x - 3x^2$ at $x = 1$.

For this problem, use the definition of the derivative at a point, and do not use differentiation rules such as the power, product, or quotient rules.

(6) Find the derivative of the function given by

$$s = 2t^{3/2} + 3e^2.$$

(7) Find the derivative of the function given by

$$r = 6(\sec\theta - \tan\theta)^{3/2}.$$

(8) Archimedes (287-212 B.C.) discovered that the area under a parabolic arch is two-thirds the base times the height. Sketch the parabolic arch $y = h - (4h/b^2)x^2$, $-b/2 \le x \le b/2$, assuming that h and b are positive. Then use calculus to find the area of the region enclosed between the arch and the x-axis.

(9) A girl flies a kite at a height of 300 ft, the wind carrying the kite horizontally away from her at a rate of 25 ft/sec. How fast must she let out the string when the kite is 500 ft away from her?

(10) Find the extreme values (absolute and local) of the function

$$y = x \ln x$$

over its natural domain, and where they occur.

(11) Graph the function

$$y = x^2 + \frac{2}{x}$$

according to the following instructions taken from the book:

Procedure for graphing $y = f(x)$:

(a) Identify the domain of f and any symmetries the curve may have.
(b) Find the derivatives y' and y''.
(c) Find the critical points of f, if any, and identify the function's behavior at each one.
(d) Find where the curve is increasing and where it is decreasing.
(e) Find the points of inflection, if any occur, and determine the concavity of the curve.
(f) Identify any asymptotes that may exist.
(g) Ploy key points, such as the intercepts and the points found in steps (c)-(e), and sketch the curve together with any asymptotes that may exist.

(12) A rectangle has its base on the x-axis and its upper two vertices on the parabola $y = 12 - x^2$. What is the largest area the rectangle can have, and what are its dimensions?

(13) Evaluate

$$\int \left(8y - \frac{2}{y^{1/4}}\right) dy.$$

(14) Evaluate

$$\int_1^4 \frac{10\sqrt{v}}{(1 + v^{3/2})^2}\, dv.$$

(15) Find the area of the region enclosed by

$$y = x^2 - 2, \quad y = 2.$$

(16) You remove the cap of a sphere of radius r. Assume that the cap has height $h < r$ and that its base is a circle. Find the volume of the cap and the volume of the remaining portion of the sphere.

You may, if you wish, use the formula for the volume of a sphere without explaining why it is true.