Final Examination (version 2) - Math 141, Frank Thorne (thorne@math.sc.edu)

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Please work without books, notes, calculators, or any assistance from others. Please **show all your work, explain yourself clearly, draw pictures where appropriate, and put equals signs where they belong.**

If you have any questions, feel free to ask me. Please do your work on separate paper; you should staple this sheet to your work (put this on top) and turn in everything together.

Each problem is worth 10 points; a total of 160 points is possible. **GOOD LUCK!**

(1) What does the Fundamental Theorem of Calculus say? Explain carefully and thoroughly.

You do not have to explain why it is true. But you must explain both parts.

(2) What does the first derivative tell you about the shape of a graph?

(3) Compute

$$\lim_{x \to -3} (x^2 - 13).$$

(4) Refer to Figure 1 for a graph and a four-part question concerning the graph.

(5) Find the slope of the curve $y = \frac{x-1}{x+1}$ at $x = 0$.

For this problem, use the definition of the derivative at a point, and do not use differentiation rules such as the power, product, or quotient rules.

(6) Find y'' if

$$y = \left(1 + \frac{1}{x}\right)^3.$$

(7) Sketch the graph of the function $\tan^{-1}(x)$. State its domain and range, and compute its derivative.

It is not enough to have memorized the derivative, you must explain your answer. You may assume that $0 < x < \pi/2$ *if this is helpful.*

(8) The marginal cost of printing a poster when x posters have been printed is

$$\frac{dc}{dx} = \frac{1}{2\sqrt{x}}$$

dollars. Find $c(100) - c(1)$, the cost of printing posters $2 - 100$.

(9) Two commercial airplanes are flying at an altitude of $40,000$ ft along straight-line courses that intersect at right angles. Plane A is approaching the intersection point at a speed of 442 knots (nautical miles per hour; a nautical mile is 2000 yd). Plane B is approaching the intersection at 481 knots. At what rate is the distance between the planes changing when A is 5 nautical miles from the intersection point and B is 12 nautical miles from the intersection point?

Since you don't have access to a calculator, you are welcome to approximate messy numbers. If you choose to do so, explain clearly what you are doing.

(10) Find the extreme values (absolute and local) of the function

$$y = \frac{x+1}{x^2+2x+2}$$

over its natural domain, and where they occur.

(11) Graph the function

$$y = \sqrt{16-x^2}$$

according to the following instructions taken from the book:

Procedure for graphing $y = f(x)$:

(a) Identify the domain of f and any symmetries the curve may have.
(b) Find the derivatives y' and y''.
(c) Find the critical points of f, if any, and identify the function's behavior at each one.
(d) Find where the curve is increasing and where it is decreasing.
(e) Find the points of inflection, if any occur, and determine the concavity of the curve.
(f) Identify any asymptotes that may exist.
(g) Ploy key points, such as the intercepts and the points found in steps (c)-(e), and sketch the curve together with any asymptotes that may exist.

(12) A 1125 ft^3 open-top rectangular tank with a square bas x ft on a side and y ft deep is to be built with its top flush (i.e., level) with the ground to catch runoff water. The costs associated with the tank involve not only the material from which the tank is made but also an excavation charge proportional to the product xy.

(a) If the total cost is

$$c = 5(x^2 + 4xy) + 10xy,$$

what values of x and y will minimize it?

(b) Give a possible scenario for the cost function in part (a).

(13) Evaluate

$$\int (1 - x^2 - 3x^5)\, dx.$$

(14) Evaluate

$$\int \frac{1}{x^3}\sqrt{\frac{x^2-1}{x^2}}\, dx.$$

(15) Find the area of the shaded region in Figure 2.

(16) Find the volume of the solid generated by revolving the region bounded by the following lines and curves about the x-axis:

$$y = 2\sqrt{x}, \quad y = 2, \quad x = 0$$