Math 554.01 - Analysis I
Test 2 – October 25, 2001

1	(30 *pts*)
2	(10 *pts*)
3	(10 *pts*)
4	(15 *pts*)
5	(15 *pts*)
6	(15 *pts*)
7	(5 *pts*)

Name: ____________________________

Directions: To receive credit, you must justify your statements unless otherwise stated. Answers should be provided in complete sentences.

1. [**Warmup**] Give an example of each of the following for the metric space of real numbers (you do not need to justify).

   (a) an open set which is not an open interval.

   (b) an infinite closed set which is not a closed interval.

   (c) a set which is closed, but has no limit points.

   (d) a set which is open, but has no limit points.

   (e) a sequence which is bounded, but is not convergent.

   (f) a sequence which is convergent, but is not monotone.

2. Using the definition of "convergence of a sequence," prove that if $\{b_n\}$ converges to b ($b \neq 0$), then $\{\frac{1}{b_n}\}$ converges to $\frac{1}{b}$.

3. Using the **properties** of limits, determine whether or not the following limit exists. Be sure to state which property you are using as you show your work.

$$\lim_{n\to\infty} \frac{1+\sqrt{n}}{3-n}.$$

4. a.) Give the definition of an **open ϵ-neighborhood** of a real number x_0.

b.) Give the definition of an **open** set of real numbers.

c.) Prove that intersection of a finite number of open sets is open.

5. a.) Define **limit point** for a set C of real numbers.

b.) Define "limit of a function at a point x_0."

c.) Using the definition, prove that $\lim\limits_{x \to \frac{1}{4}} \sqrt{x} = \frac{1}{2}$.

6. a.) Give the definition for a function f to be continuous at a point x_0.

b.) State an equivalent condition (involving sequences) in order to verify that a function is continuous at x_0.

c). Using properties of limits and part b), show that $f(x) = \dfrac{x^2+1}{\sqrt{x}+2}$ is continuous at $x_0 = 2$.

7. Negate the statement that a function is continuous at a point.