Math 554- 703 I - Analysis I
Homework Assignment # 9
Due Tuesday - Nov. 20, 2001

1. A set $\tilde{C}$ is called *relatively closed with respect to* A if its complement with respect to A is relatively open with respect to A.

   (a) Prove that a subset $\tilde{C}$ of a set A of real numbers is relatively closed with respect to A if and only if there exists a closed set C of real numbers such that $\tilde{C} = C \cap A$.
   [Hint: Recall from lecture that a subset $\tilde{O}$ of a space A is relatively open with respect to A if and only if there exists an open subset O of R so that $\tilde{O} = O \cap A$.]

   (b) Prove that a function $f : A \to B$ is continuous if and only if for each relatively closed set C in B, the set $f^{-1}(C)$ is relatively closed in A.

2. Determine all compact connected subsets of real numbers. Justify your answer.

3. Suppose that f and g are continuous real-valued functions on the interval $[a, b]$ which satisfy $g(a) \leq f(a)$ and $f(b) \leq g(b)$, then prove that there exists a number $c \in [a, b]$ such that $f(c) = g(c)$.

4. **Extra Credit**. Suppose that f is a continuous function that maps the interval $[a, b]$ into itself, then prove that f has a fixed point: i.e. that there exists a number c in the interval so that $f(c) = c$.