Math 554- 703 I - Analysis I
Homework Assignment # 1
Due Thursday - August 27, 2001

1. Using the field axioms, prove that the multiplicative identity is unique.

<u>proof</u> Let e_1 & e_2 be multiplicative identities, then

e_2 is an identity

$$e_1 = e_2 e_1 = e_2 \quad \blacksquare$$

e_1 is a multiplicative identity

2. Using the field axioms, prove that for each $a \in F, a \neq 0$, the multiplicative inverse of a is unique.

<u>proof</u> Suppose b & c are both multiplicative inverses of a, $a \neq 0$, then

$$b = b \cdot 1 = b(ac) = (ba)c = 1 \cdot c = c$$

since 1 is the multiplicative identity, $ac = 1$, the associative property, $ba = 1$, and 1 is the multiplicative identity all hold.

3. Suppose that F is an ordered field, i.e. a field with a positive cone. Suppose that $a < b$ and $0 < c$, then prove that $ac < bc$.

<u>proof</u> Since $a < b$, then $(b-a) \in \mathbb{P}$. By definition $0 < c$ implies $c \in \mathbb{P}$. $\mathbb{P}$ is a positive set so the product of two positive elements is positive:

$$(b-a) \cdot c \in \mathbb{P}.$$

But then

$$(bc) - (ac) \in \mathbb{P}.$$

By definition of $<$, it follows that

$$ac < bc. \quad \blacksquare$$