Math 554.01 - Analysis I
Test 2 – March 5, 2004

1	(15 *pts*)
2	(20 *pts*)
3	(20 *pts*)
4	(20 *pts*)
5	(15 *pts*)
6	(10 *pts*)

Name: ____________________________ **4 Digit CODE:** ______

Directions: To receive credit, you must justify your statements unless otherwise stated. Answers should be provided in complete sentences.

1. a.) Define *metric*.

   b.) Give two examples of metric spaces. (You do not need to verify the properties.)

2. Let (E, d) be a metric space.
   a.) Define *open set*.

   b.) Prove that an open ball is an open set.

   c.) Let d be the discrete metric on a set E. Prove that each subset S of E is a closed set.

3. a.) Give the definition of a closed set.

   b.) Give the definition of a limit point of a set.

   c.) Prove that a set is closed if and only if it contains all its limit points.

4. Using the **definition** of "convergence of a sequence," prove that
   a.) $\{a_n\}$ converges to a implies that a_n^2 converges to a^2.

   b.) $\{a_n\}$ converges to a implies that $|a_n|$ converges to $|a|$.

5. Using the **properties** of limits, determine whether or not the following limit exists. Be sure to state which property you are using as you show your work.

a.) $a_n = 1 - \dfrac{2}{n}$

b.) $b_n = 2 + \dfrac{3}{n^2}$

c.) Consider the sequence, $c_n = \dfrac{n-1}{2n^2+3}$. Use parts a.) and b.) to determine the convergence of $\{c_n\}$.

6. Suppose that E is a metric space and $S \subset E$ is complete. Prove that S is closed.