

MATH 554.01 - ANALYSIS I
TEST 1 – SEPTEMBER 19, 1997

1	(20pts)
2	(10pts)
3	(15pts)
4	(15pts)
5	(15pts)
6	(15pts)
7	(10pts)

Name: _____

Directions: To receive credit, you must justify your statements unless otherwise stated. Answers should be provided in complete sentences.

1. a.) Prove $(-1)(a) = (-a)$.

b.) Prove that $a > 0$ implies $(-a) < 0$.

2. Negate the statement “there exists a positive number δ such that for each natural number m there is a natural number k larger than m so that $|f_k - f_m| < \delta$.”

3. a.) Define countability.

b.) Sketch a proof that the set of rational numbers is countable.

4. Let A be a nonempty subset of \mathbb{R} .

a.) Define 'upper bound' for A .

b.) Define 'least upper bound' for A .

c.) Prove that if $\beta = \text{l.u.b. } A$, then

β is an upper bound for A ,

and

if $\epsilon > 0$, then there exists $a \in A$ such that $\beta - \epsilon < a$.

5. a.) Prove that \mathbb{N} is not bounded.

b.) State and prove the Archimedean principle.

6. a.) Define $\lim_{n \rightarrow \infty} a_n = L$.

b.) Prove $\lim_{n \rightarrow \infty} (2n + 1)^{-1} = 0$.

7. Suppose that $\{a_n\} \rightarrow a$, then prove that $\{|a_n|\} \rightarrow |a|$.