Solutions for Homework #7
Math 554

1. Polynomials are continuous.

<u>Soln</u> Products of continuous functions are continuous, so $f(x) = x \cdot x$ is continuous. By induction, $f(x) = x^n$ is continuous, $\forall n \in \mathbb{N}$. Hence $a_n x^n$ is continuous $\forall n \in \mathbb{N}$. Since finite sums of continuous functions are continuous, then

$$P(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$$

is continuous.

2. Rational functions are continuous on their domains.

<u>Soln</u>. Let $R(x) = P(x)/Q(x)$ where P & Q are polynomials. $\mathrm{dom}(R) = \{x \in \mathbb{R} \mid Q(x) \neq 0\} = \mathbb{R} \setminus \{\text{zeroes of } Q\}$. But quotients of continuous functions are continuous except where they are undefined.

3. Suppose $f: A \to B$ & $g: C \to D$ are continuous, then $g \circ f$ is continuous.

<u>Soln</u> Let $\tilde{O}$ be open in D, then $g^{-1}(\tilde{O}) := O$ is open in C. f continuous $\Rightarrow$ $f^{-1}(O)$ is open in A. But then

$$(g \circ f)^{-1}(\tilde{O}) = f^{-1}(g^{-1}(\tilde{O}))$$

is open, for each open set $\tilde{O}$ & $f \circ g$ must be continuous.

4. $g(y) = \sqrt{y+1}$, $f(x) = \dfrac{x-1}{x^2-3x+2}$. Find $\mathrm{dom}(g \circ f)$.

<u>Soln</u> $\mathrm{dom}(g) = [-1, \infty)$, $\mathrm{dom}(f) = \mathbb{R} \setminus \{1, 2\}$.

If $x \neq 1, 2$, then $f(x) = \dfrac{1}{x-2} \geq -1 \iff \begin{cases} \underline{\text{Case 1}} \;\; x > 2: (1 \geq 2-x) \;\&\; (x \neq 1, 2) \\ \qquad\qquad \text{or } (x \geq 1) \;\&\; (x \neq 1, 2) \\ \underline{\text{Case 2}} \;\; x < 2: (1 \leq 2-x) \;\&\; (x \neq 1, 2) \\ \qquad\qquad \text{or } (x \leq 1) \;\&\; (x \neq 1, 2). \end{cases}$

$$\iff x \in (2, \infty) \cup (-\infty, 1)$$

$$\mathrm{dom}(g \circ f) = (-\infty, 1) \cup (2, \infty).$$