

MATH 554/703I - ANALYSIS I
TEST 3 SAMPLE PROBLEMS
APRIL 14, 2004

Here are 10 problems to provide examples from all areas covered since Test 2. The test on Wednesday will have 7-8 problems.

Directions: To receive credit, you must justify your statements unless otherwise stated. Answers should be provided in complete sentences.

1. Give an example of each of the following and (very) briefly justify your answer:
 - (a) A bounded set of real numbers that is not compact.
 - (b) A connected set of real numbers that is not compact.
 - (c) A real-valued continuous function that does not satisfy the Extreme Value Theorem.
 - (d) A real-valued continuous function that does not satisfy the Intermediate Value Theorem.
2. a.) Spell 'Bolzano-Weierstrass'.
b.) State the Bolzano-Weierstrass property for a subset of real numbers.
3. a.) Define **open cover** for a set.
b.) Define what it means for a set to be **compact**.
c.) Suppose K is compact and $f : K \rightarrow \mathbb{R}$ is continuous. Prove that $f[K]$ is compact.
4. State and sketch a proof of the **Heine-Borel theorem**.
5. Prove that each closed and bounded subset of real numbers is a compact set.
6. a.) Define what it means for a set A to be a connected subset of real numbers.
b.) Show that a subset A of real numbers is connected implies that A is an interval.
7. State and prove the **Intermediate Value Theorem**
8. a.) Define what it means for f to be differentiable at $x = x_0$.
b.) Prove that f is continuous at $x = x_0$ if f is differentiable at $x = x_0$.
9. Show that the function $f(x) = x^5$ is **one-to-one** from $[0, \infty)$ **onto** $[0, \infty)$. Prove that its inverse is continuous.
10. Pick one of the following:
 - a.) State and prove the Chain Rule.
 - b.) State and prove the Quotient Rule