Math 554/703I - Analysis I

HW #7 – Due March 26, 2004

Defn. Given $f : A \to B$ and $g : C \to D$, the composition $g \circ f$ is defined by $g \circ f(x) = g(f(x))$, where $x \in \text{dom}(f)$ and $f(x) \in \text{dom}(g)$.

1. Using the properties of sequential limits, carefully prove that any polynomial of degree n, $P(x) = a_0 + a_1 x + \ldots a_n x^n$ is a continuous function. Here the coefficients $a_j \in \mathbb{R}, j = 1, \ldots, n$.
2. Using the properties of sequential limits, carefully prove that any rational function $R(x) = \frac{P(x)}{Q(x)}$, where P and Q are polynomials, is continuous on its domain.
3. If $A, B, C, D \subset \mathbb{R}$ and $f : A \to B$ and $g : C \to D$ are both continuous functions, then prove that $g \circ f$ is continuous.
4. Detemine the domain of $h(x) := g \circ f(x)$ if $g(y) = \sqrt{y+1}$ and $f(x) = \frac{x-1}{x^2-3x+2}$.