Math 554 HW-6

No.

Solutions

Date . .

1. <u>Proof</u>: Suppose S is not closed. The CS is not open. So $\exists\ p \in CS$ such that $\forall$ open ball $B_\varepsilon(p)$ contains points of S. So $\forall\ n \in \mathbb{N}$ $\exists\ p_n \in S$ such that $d(p, p_n) < \frac{1}{n}$. Then this follows that $\lim_{n\to\infty} p_n = p$ with every $p_n \in S$ and $p \notin S$. This is a contradiction.
$\therefore$ S is closed.

2. <u>Proof</u>:
$$\left|\frac{1}{b} - \frac{1}{b_n}\right| = \frac{|b_n - b|}{|b|\cdot|b_n|}.$$

Given $\varepsilon > 0$ $\exists\ N \in \mathbb{N}$ such that
$|b - b_n| < \min\{\frac{|b|}{2}, \frac{|b|^2\varepsilon}{2}\}$ for all $n > N$

Then we have
$$|b_n| = |b - (b - b_n)| \geq |b| - |b - b_n| > |b| - \frac{|b|}{2} = \frac{|b|}{2} \quad \text{for all } n > N$$
$$\therefore \left|\frac{1}{b} - \frac{1}{b_n}\right| = \frac{|b - b_n|}{|b|\cdot|b_n|} < \frac{|b|^2\frac{\varepsilon}{2}}{|b|\cdot(\frac{|b|}{2})} = \varepsilon$$

i.e. $\lim_{n\to\infty} \frac{1}{b_n} = \frac{1}{b}$