Math 554 HW #5

No.

Date

Solution

1. a **Proof:** ~~Let~~ Let $\{C_i\}_{i=1}^{n}$ be finite number of closed sets

so $\bigcup_{i=1}^{n} C_i = \bigcup_{i=1}^{n} ((C_i^c)^c = (\bigcap_{i=1}^{n} C_i^c)^c$

Clearly $\bigcap_{i=1}^{n} C_i^c$ is finite intersection of open sets and it is open. so $(\bigcap_{i=1}^{n} C_i^c)^c$ is closed.

i.e $\bigcup_{i=1}^{n} C_i$ is closed.

b. **Proof:** Let $\{C_\alpha\}_{\alpha\in A}$ be collection of closed sets.

$\bigcap_{\alpha\in A} C_\alpha = \bigcap_{\alpha\in A} (C_\alpha^c)^c = (\bigcup_{\alpha\in A} C_\alpha^c)^c$

Notice $\bigcup_{\alpha\in A} C_\alpha^c$ is open since it is the union of open sets $\{C_\alpha^c\}$. So the complement of $\bigcup_{\alpha\in A} C_\alpha^c$ is closed.

i.e $\bigcap_{\alpha\in A} C_\alpha$ is closed.

2. **Proof:** $\because$ $d(p-q) = \begin{cases} 1 & \text{if } p\neq q \\ 0 & \text{if } p=q \end{cases}$ for all $p, q \in E$

① We want to show $\forall\ S \subset E$ is open.

$\forall\ p\in S$ $\exists\ \delta = \frac{1}{2} > 0$ such that $B_\delta(p) = \{p\} \subseteq S$

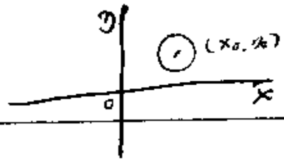
so $S \subset E$ is open.

② We want to show $\forall S \subset E$ is closed

~~$\forall\ p\in S$ $\forall\ \epsilon>0$ such that B~~

By ① we can prove S^c is open.

so $(S^c)^c = S$ is closed.



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3. Proof: ~~$\forall\ p_0 \in ((x_0, y_0))$~~ Let $S = \{(x,y) \in \mathbb{R}^2 \mid y > 0\}$

$\forall\ p_0 \in S, \left((x_0, y_0)\right)$, $y_0 > 0$

$\therefore$ $B_{\frac{y_0}{2}}(p_0)$ is entirely contained in S.

$\therefore$ S is open.