Math 554/703I - Analysis I
HW #5 – Due Monday, February 23, 2004

In each problem, (E, d) is a metric space.

1. Show that in E
   a. the union of any finite number of closed sets is a closed set.
   b. the intersection of any collection of closed sets is a closed set.

2. If the metric for E is the **discrete metric**, then show that each set $S \subset E$ is always open and closed.

3. Show that the upper half plane
$$\left\{(x, y) \in \mathbb{R}^2 | y > 0\right\}$$
is an open set when d is the Euclidean metric (i.e. d_2).

4. (Grad Students and E.C.) Show that
$$\left\{(x, y) \in \mathbb{R}^2 | x > y + 1\right\}$$
is an open subset of the plane equipped with the standard Euclidean metric.

5. Suppose $S \subset E$, then prove that p_0 is a limit point of S if and only if each open ball in E which has p_0 as its center contains an infinite number of points from S.

6. Prove that any finite subset S of E is closed and each point of S is an isolated point. (Defn: a point p_0 is called an *isolated point* of S if $p_0 \in S$ and there is an open ball $B_\epsilon(p_0)$ which contains no other points of S.)