Math 554/703I - Analysis I
HW #4 – Due February 13, 2004

1. For a set E, consider the **discrete metric** given by

$$d(p,q) := \begin{cases} 0, & \text{if } p = q \\ 1, & \text{otherwise.} \end{cases}$$

   Prove that (E,d) is a metric space.

2. Let

$$\phi(x) := \frac{x}{1+x}.$$

   Assuming that $\phi(a+b) \leq \phi(a) + \phi(b)$ and that $\phi(a) \leq \phi(b)$, when $0 \leq a \leq b$, prove that $d(p,q) := \phi(|p-q|)$ is a metric for the real numbers.
   (Verify on your own the two assumptions on the function ϕ.)

3. For vectors in the plane $\mathbb{R}^2$, consider the **city block metric** given by

$$d(\mathbf{x},\mathbf{y}) := \sum_{j=1}^{2} |x_j - y_j|$$

   where $\mathbf{x} = (x_1, x_2)$ and $\mathbf{y} = (y_1, y_2)$. Show that this is in fact a metric. Can you see how to extend the definition to higher dimensions, i.e. for $\mathbb{R}^d$?

4. Consider the set E of real-valued functions on $[a,b]$ and define the **uniform metric** d on E by

$$d(f,g) := \mathbf{lub}\left\{|f(x) - g(x)| : a \leq x \leq b\right\}.$$

   Show that d is in fact a metric.